School of Electronic and Communications Engineering

Part I: Operational Amplifiers & Their Applications

Contents

- Opamps fundamentals
- Opamp Circuits
  - Inverting & Non-inverting Amplifiers
  - Summing & Difference Amplifiers
  - Integrators & Differentiators
- Opamp Applications:
  - Rectifiers
  - Comparators & Relaxation Oscillators
  - Feedback (sine-wave) Oscillators

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Administrative matters

- Recommended textbook:

  A.S. Sedra and K.C. Smith: “Microelectronic Circuits,” Oxford University Press, New York, 1998.

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Operational Amplifiers

Recommended Text:

A.S. Sedra and K.C. Smith: "Microelectronic Circuits," Oxford University Press, New York, 1998. pp. 61- 84

Contents

- Introduction
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  - Integrator
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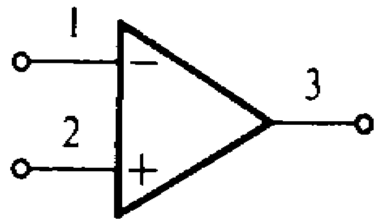
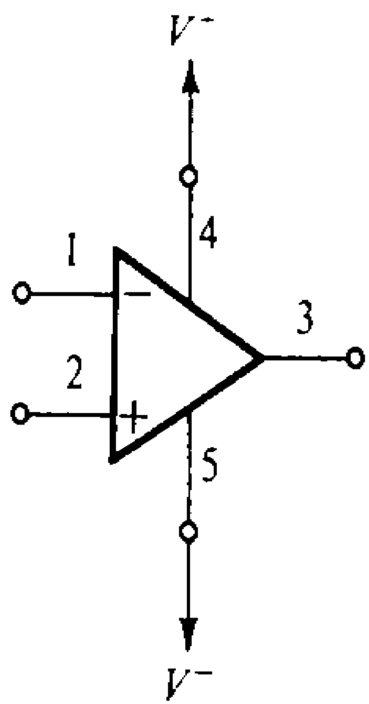
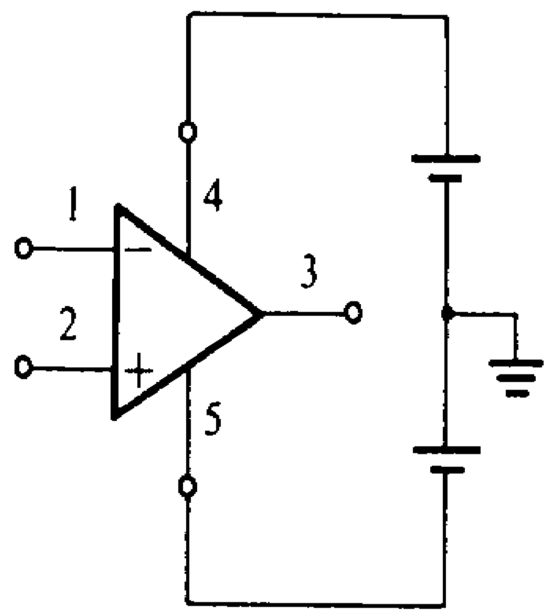
  - Weighted summer
- Noninverting configuration
  - Closed-loop gain
  - Voltage follower

Introduction-Basics



Circuit symbol for the op amp.





The op amp shown connected to dc power supplies.

Introduction-Basics

- Basic Configuration of Op amps
  - Three terminals (2 inputs and 1 output)
  - Bipolar power supply
  - Reference ground is the "power supply ground"

Introduction-Basics

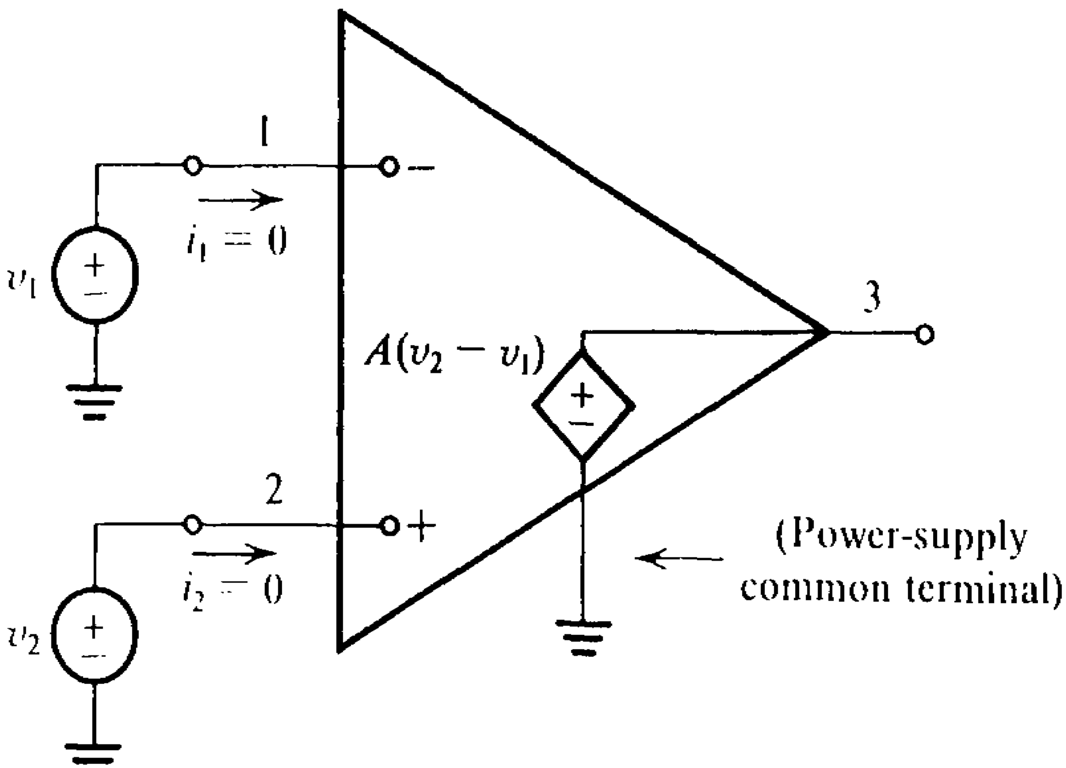


Fig. 2.3 Equivalent circuit of the ideal op amp.

Introduction-Basics

- Characteristics of Opamps

Output voltage is proportional to the difference between the two inputs.

$$V_3 = A(V_2 - V_1)$$

Introduction-Ideal Op amps

- The open-loop gain *A* is very very large, and can be considered as infinite
- The two input terminals are "virtually open".
- That is, the input impedance of the terminals are very very big (infinite).
- The output impedance is very very small and can be considered as zero.

Introduction- How to use OpAmps

- We would like opamps to
  - amplify the amplitude of input signal by a factor of any arbitrary value;
- However, the open-loop gain is fixed and too large, therefore some external circuits should be used to make the system close-loop;
- There are two configurations in terms of using the external circuits:
  - Non-inverting and
  - Inverting.

Inverting configuration

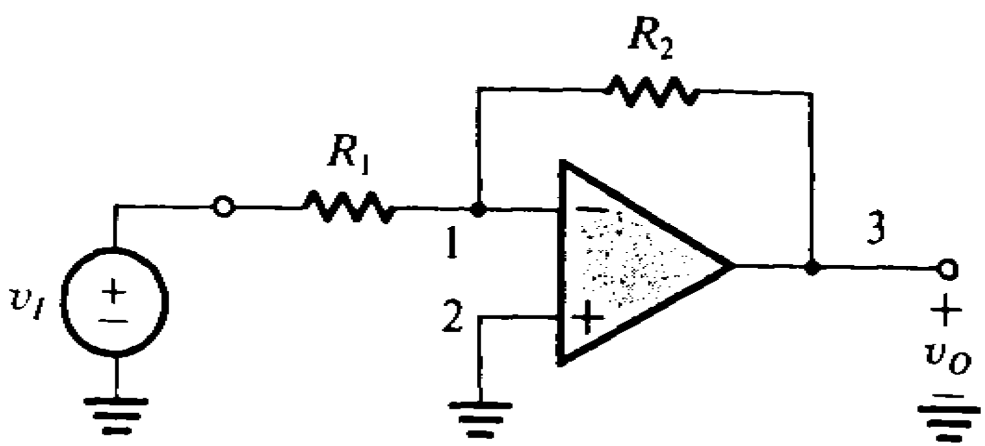
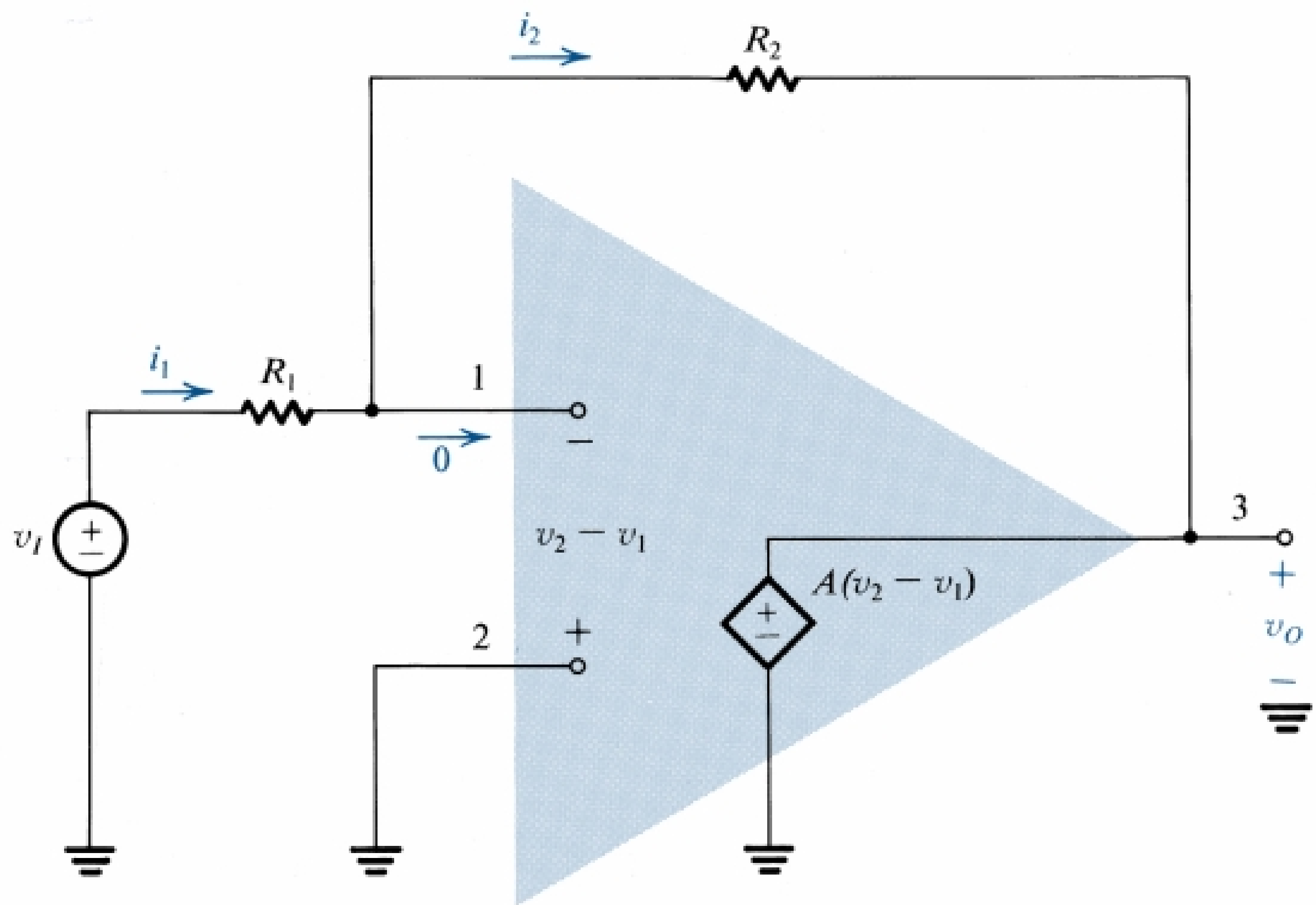


Fig. 2.4 The inverting closed-loop configuration.

$$G \equiv \frac{v_O}{v_I}$$

Inverting configuration



(a)

Inverting close-loop configuration

- The gain A is very large (ideally infinite), If we assume that the circuit is "working" and producing a finite output voltage at terminal 3, then the voltage between the op amp input terminals should be negligibly small.
- Specifically, if we call the output voltage v_o, then, by definition.

$$v_2 - v_1 = \frac{v_o}{A}$$

- voltage at the inverting input terminal (v_1) is given by $v_1=v_2$. That is, because the gain A approaches infinity, the voltage v_1 approaches v_2. We speak of this as the two input terminals "tracking each other in potential." We also speak of a "virtual short circuit" that exists between the two input terminals.
- Here, the word *virtual* should be emphasized, and one should *not* make the mistake of physically shorting terminals 1 and 2 together while analyzing a circuit

Inverting close-loop configuration

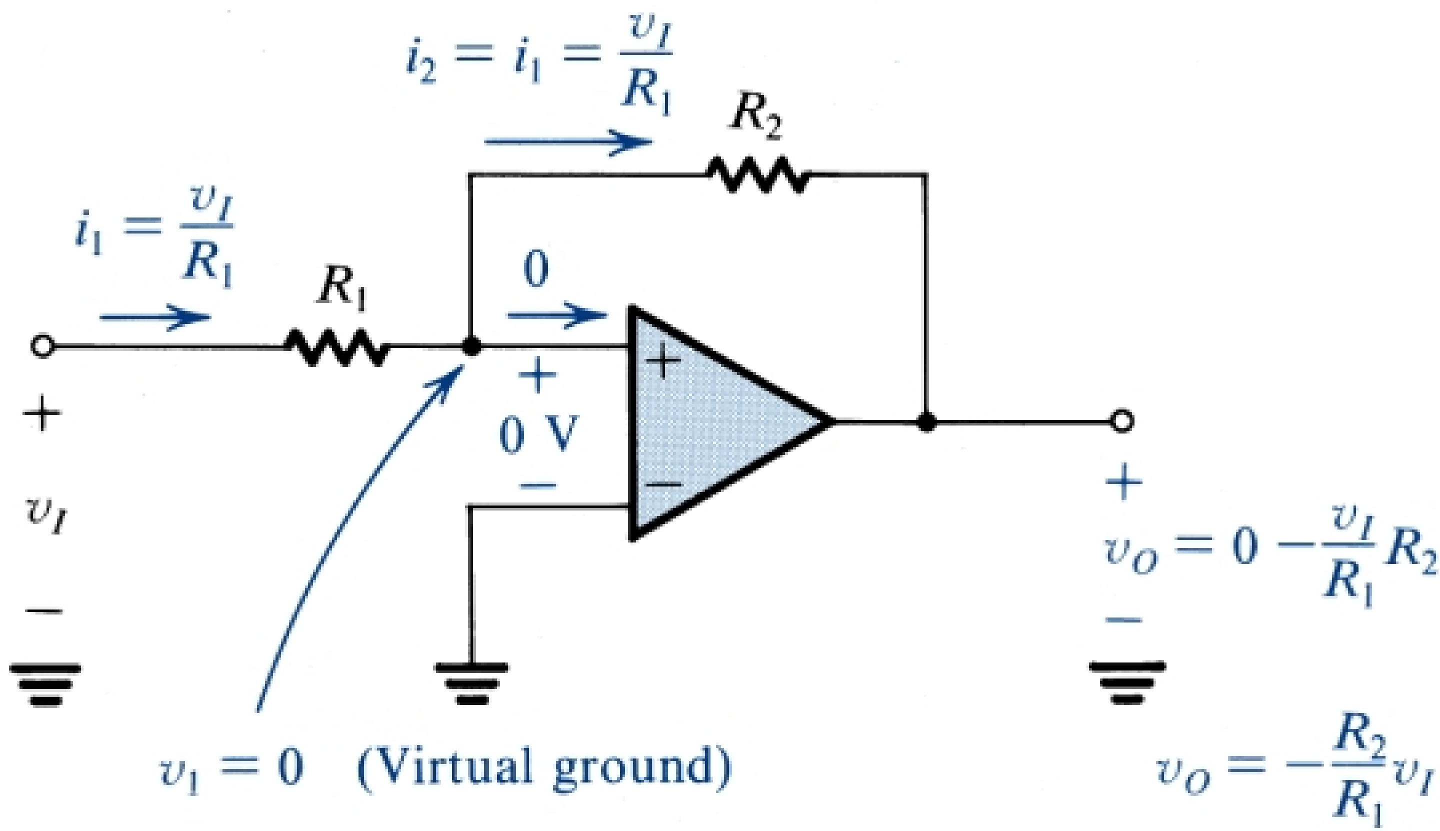
- A virtual short circuit means that whatever voltage is at 2 will automatically appear at 1 because of the infinite gain *A*. But terminal 2 happens to be connected to ground; thus v_1 = 0 and $v_2 = 0$. We speak of terminal I as being a virtual ground—that is, having zero voltage but not physically connected to ground
- Now that we have determined v_1 we are in a position lo apply Ohm's law and find *the* current i_1 through R_1 as follows:

$$i_1 = \frac{v_I - v_1}{R_1} \cong \frac{v_I}{R_1}$$

- Where will this current go? It cannot go into the op amp, since the ideal op amp has an infinite input impedance and hence draws zero current.
- It follows that it will have to flow through *R2* to the terminal 3.
- We can then apply Ohm's law to *R2* and determine v_o that is,

$$v_O = v_1 - i_1 R_1 = 0 - \frac{v_I}{R_1} R_2 \quad thus \quad \frac{v_O}{v_I} = -\frac{R_2}{R_1}$$

Inverting close-loop configuration

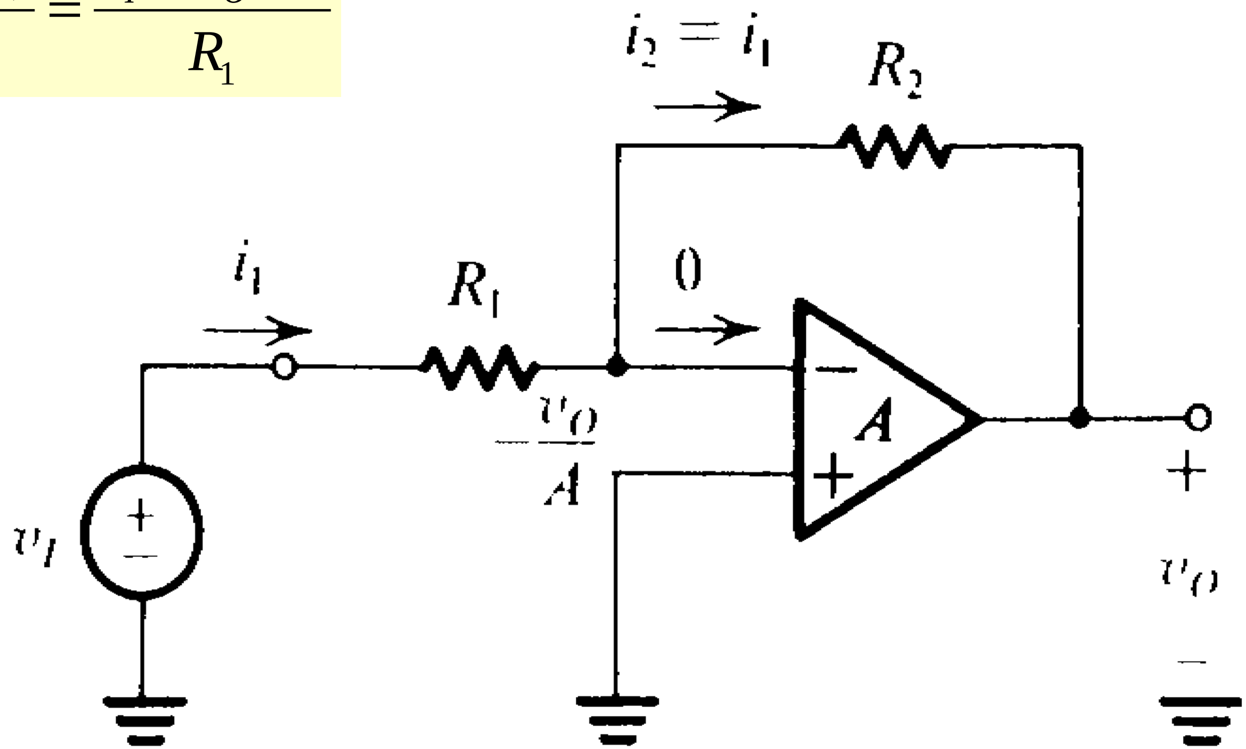


(b)

Inverting close-loop configuration (*Effect of finite open-loop gain)*

- If we denote the output voltage v_O , then the voltage between two input terminals of the opamp will be v_O /A. Since the positive input terminal is grounded, the voltage at the negative input terminal must be $-v_O /A$. the current i_I and R_I can now be found as:

$$i_I = \frac{v_I - (-v_O / A)}{R_1} = \frac{v_I + v_O / A}{R_1}$$



Inverting close-loop configuration

- The infinite input impedance of the opamp forces the current i_I to flow entirely through R_2. The output voltage v_O can thus be determined from:

$$v_O = -v_O / A - i_1 R_2 = -\frac{v_O}{A} - \left(\frac{v_I + v_O / A}{R_1} \right) R_2$$

- Collecting terms, the closed–loop gain G is found as:

$$G \equiv \frac{v_O}{v_I} = \frac{-R_2 / R_1}{1 + \left(1 + R_2 / R_1\right) / A}$$

- This is virtual ground assumption we used in our earlier analysis when the opamp was assumed to be ideal. Finally, note that this equation in fact indicates that to minimize the dependence of the closed-loop gain G on the value of open-loop gain A, we should make:

$$1 + R_2 / R_1 << A$$

Input and output resistances

Assuming an ideal op amp with infinite open-loop gain, the input resistance of the closed-loop inverting amplifier of Fig. 2.4 is simply equal to R_1. This can be seen from Fig. 2.5(b), where

$$R_i \equiv \frac{v_I}{i_1} = \frac{v_I}{v_I/R_1} = R_1$$

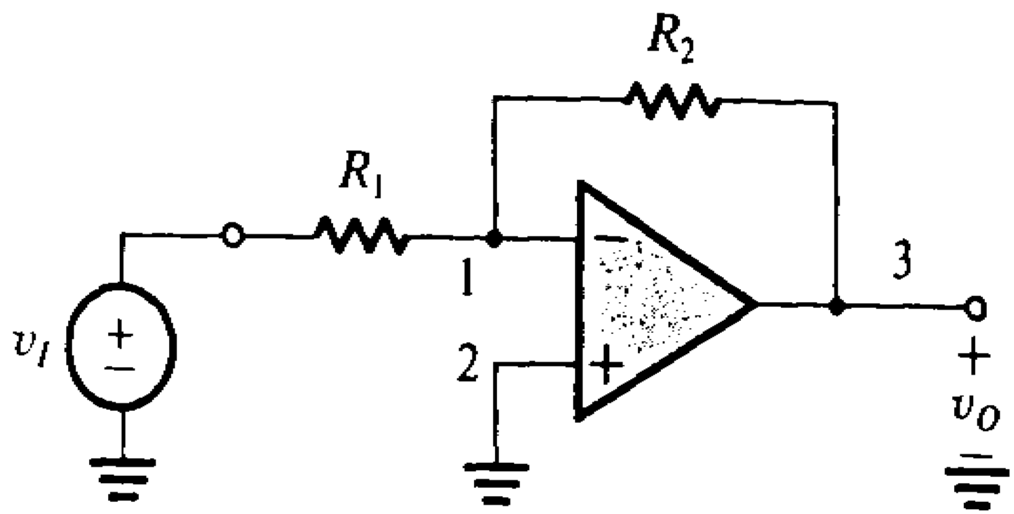


Fig. 2.4 The inverting closed-loop configuration.

$$G \equiv \frac{v_O}{v_I}$$

Input and output resistances

- Assuming an ideal op amp with infinite open-loop gain, the input resistance of the closed-loop inverting amplifier is simply equal R_1.

$$R_i \equiv \frac{v_O}{i_I} = \frac{v_I}{v_I / R_1} = R_1$$

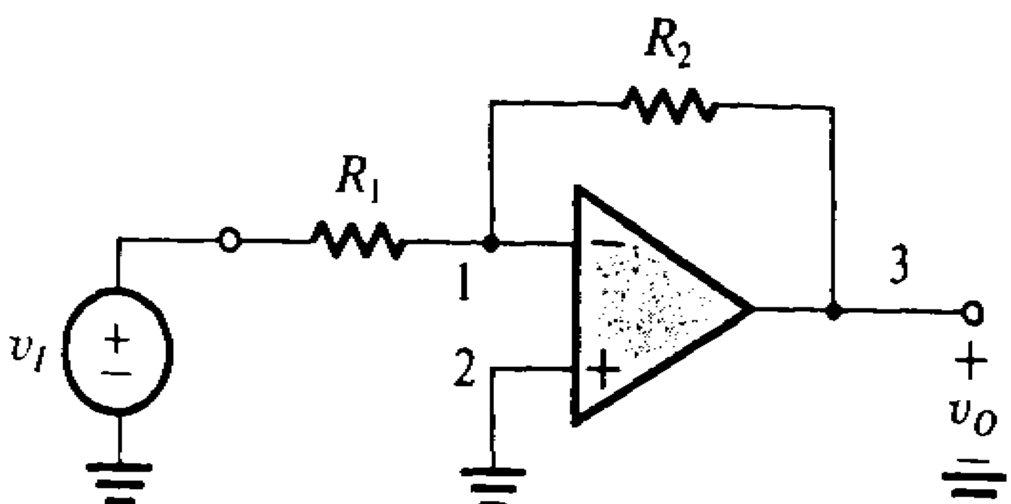
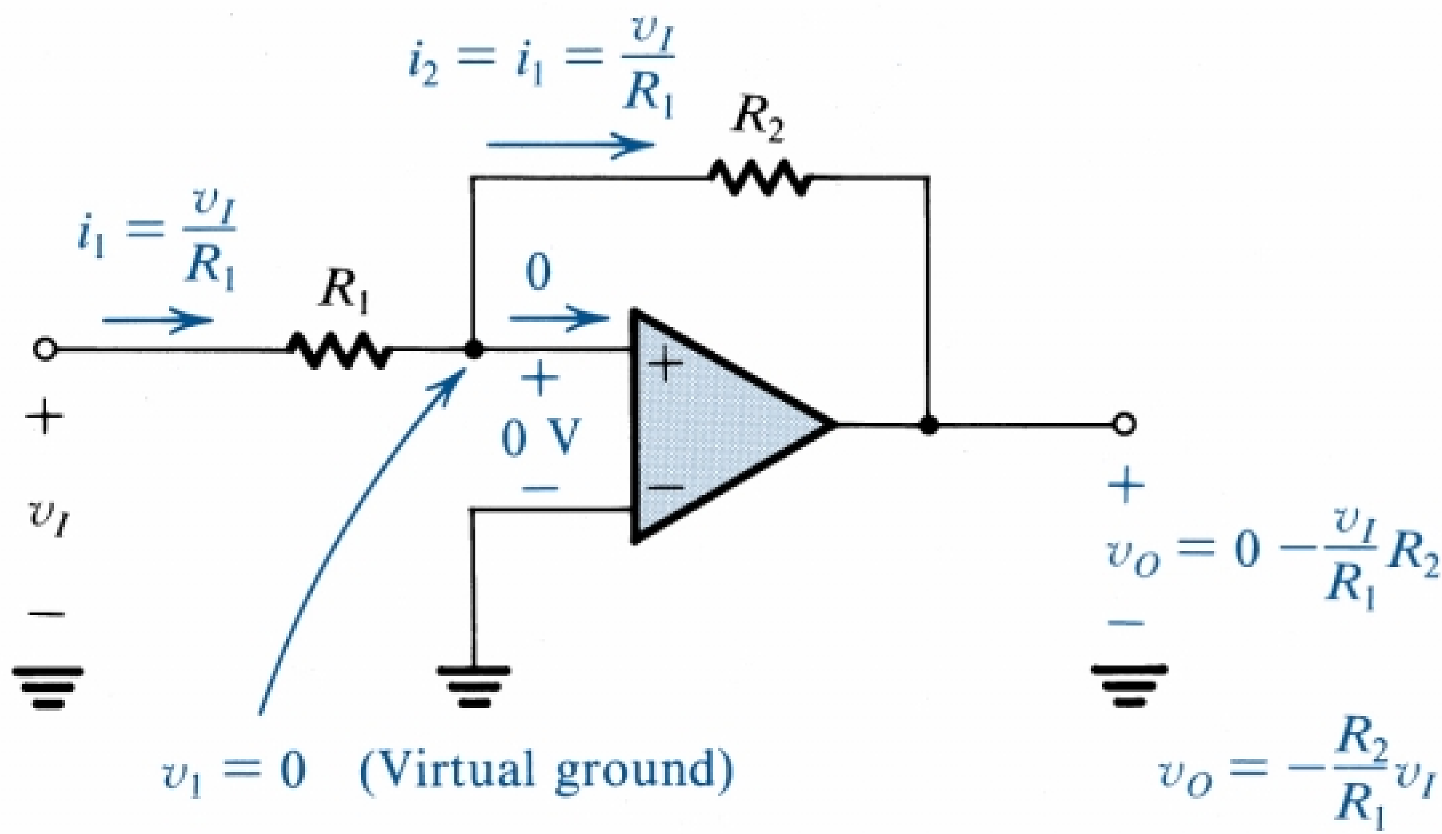


Fig. 2.4 The inverting closed-loop configuration.

$$G \equiv \frac{v_O}{v_I}$$

Input and output resistances

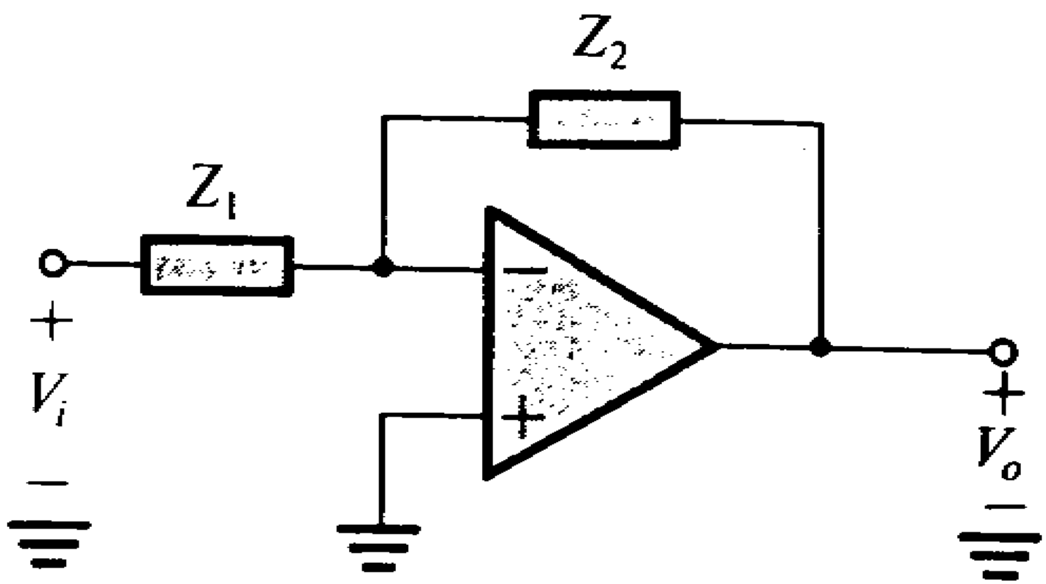


(b)

General impedances

Consider first the generalized inverting configuration where impedances $Z_1(s)$ and $Z_2(s)$ replace resistors R_1 and R_2, respectively.
The resulting circuit is shown below and has the closed-loop gain or, more appropriately, the closed-loop transfer function

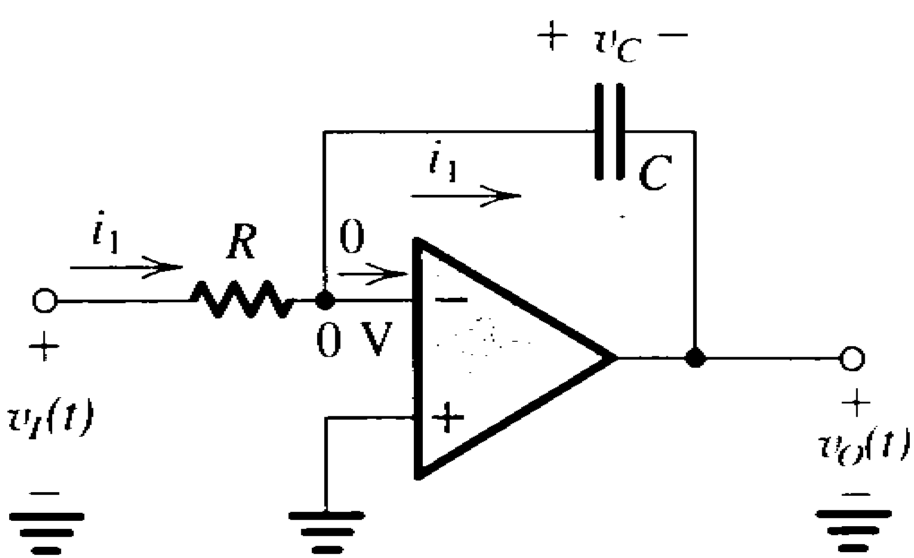
$$\frac{V_O(s)}{V_I(s)} = \frac{Z_2(s)}{Z_1(s)}$$



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

Inverting integrator
(*Miller integrator*)

- Let the input he a time-varying function $v_I(t)$. The virtual ground at the inverting op-amp input causes $v_I(t)$ to appear in effect across R, and thus the current $i_1(t)$ will be $v_I(t)/R$.
- This current flows through the capacitor C, causing charge lo accumulate on C. If we assume that the circuit begins operation at time t = 0, then at an arbitrary time t the current $i_1(t)$ will have deposited on C a charge equal to.



$$v_O(t) = -\frac{1}{CR}\int_0^t v_I(t)\,dt$$

$$\frac{V_o}{V_i} = -\frac{1}{sCR}$$

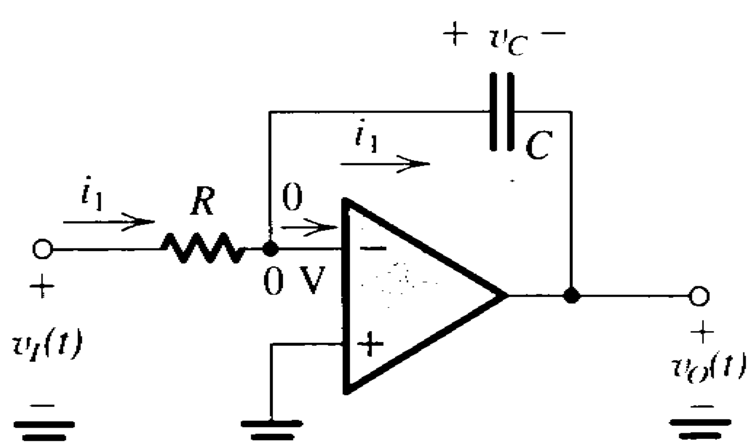
Inverting integrator
(*Miller integrator*)

Thus the capacitor voltage $v_C(t)$ will change by $\frac{1}{C}\int_0^t i_1(t)\,dt$. If the initial voltage on C (at $t = 0$) is denoted V_C, then

$$v_C(t) = V_C + \frac{1}{C}\int_0^t i_1(t)\,dt$$

Now the output voltage $v_O(t) = -v_C(t)$, thus

$$v_O(t) = -\frac{1}{CR}\int_0^t v_I(t)\,dt - V_C \qquad (2.3)$$

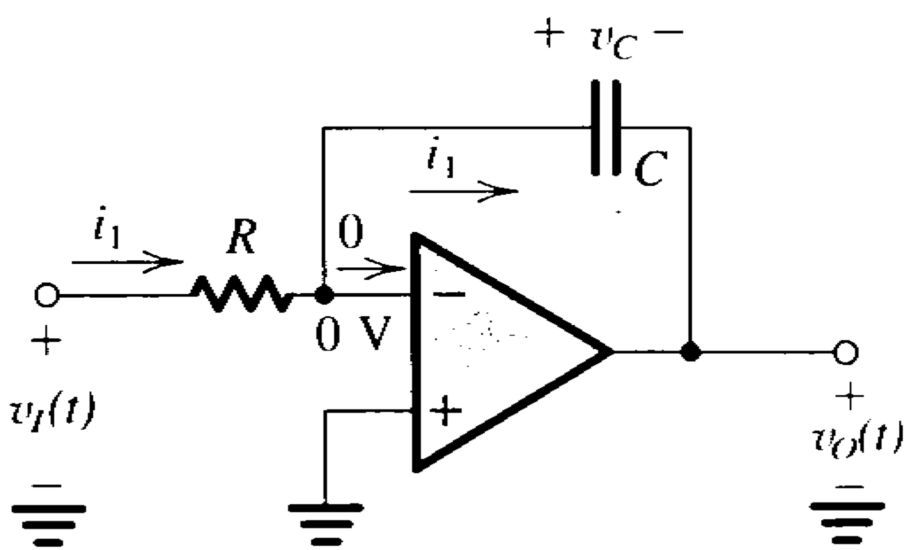


$$v_O(t) = -\frac{1}{CR}\int_0^t v_I(t)\,dt$$

$$\frac{V_o}{V_i} = -\frac{1}{sCR}$$

Inverting integrator
(*Miller integrator*)

- Thus the circuit provides an output voltage that is proportional to the time-integral of the input, with *Vc* being the initial condition of integration and *CR* is the integrator time-constant.
- Note that as expected there is a negative sign attached to the output voltage, and thus this integrator circuit is said to be an inverting integrator.
- It is also known as a Miller integrator after an early worker in this area.



$$v_O(t) = -\frac{1}{CR}\int_0^t v_I(t)\,dt$$

$$\frac{V_o}{V_i} = -\frac{1}{sCR}$$

Inverting integrator
(*Miller integrator*)

- For physical frequencies, $s = j\omega$ and

$$\frac{V_O(j\omega)}{V_I(j\omega)} = -\frac{1}{j\omega CR}$$

- Thus the integrator transfer function has magnitude

$$\left|\frac{V_O}{V_I}\right| = \frac{1}{\omega CR}$$

- and phase

$$\varphi = +90^o$$

Inverting integrator

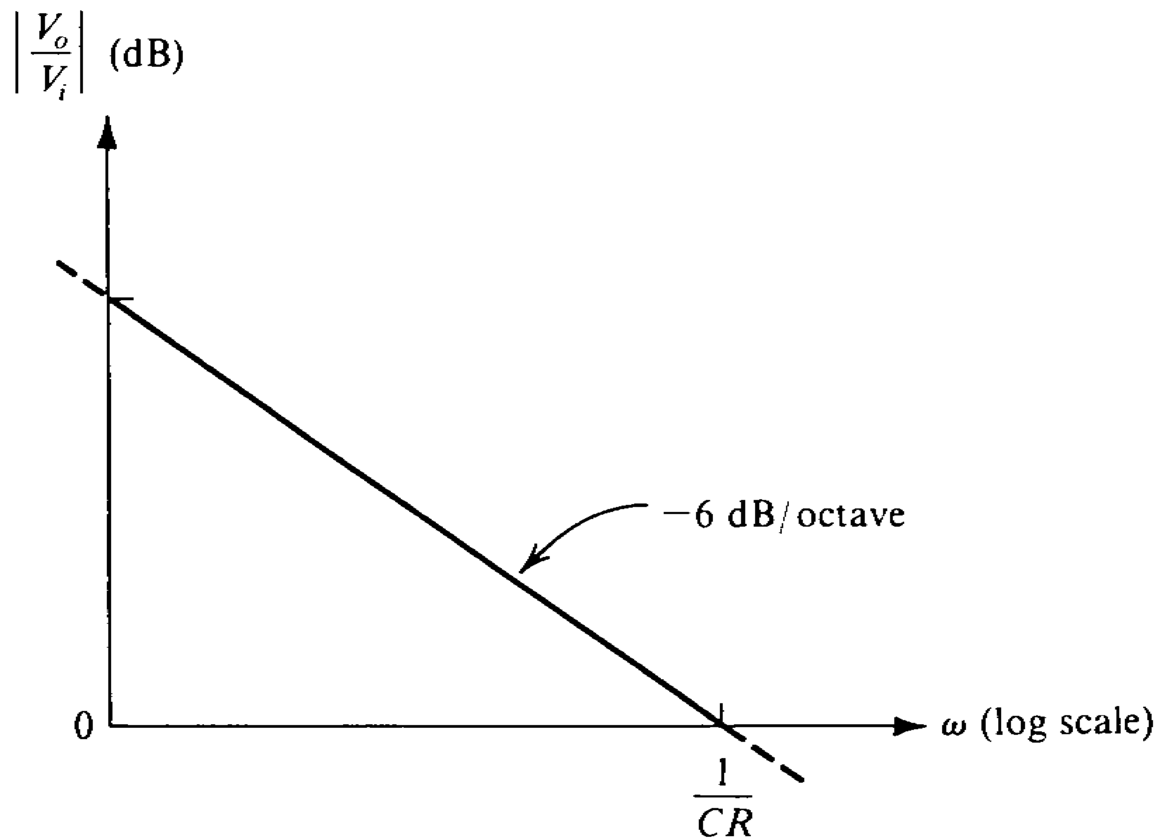
(*Miller integrator*)

- The Bode plot for the integrator magnitude response can be obtained by noting that as ω doubles (increases by an octave) the magnitude is halved (decreased by 6 dB).
- Thus the Bode plot is a straight line of slope -6 dB/octave (or equivalently, -20 dB/ decade).
- This line intercepts the 0-dB line at the frequency that makes *Vo/Vi* = 1, which) is

$$\omega_{\text{int}} = \frac{1}{CR}$$

- The frequency ω_{int} is known as the integrator frequency and is simply the inverse of the integrator rime constant.

Inverting integrator
(*Miller integrator*)



Inverting integrator
(*Miller integrator*)

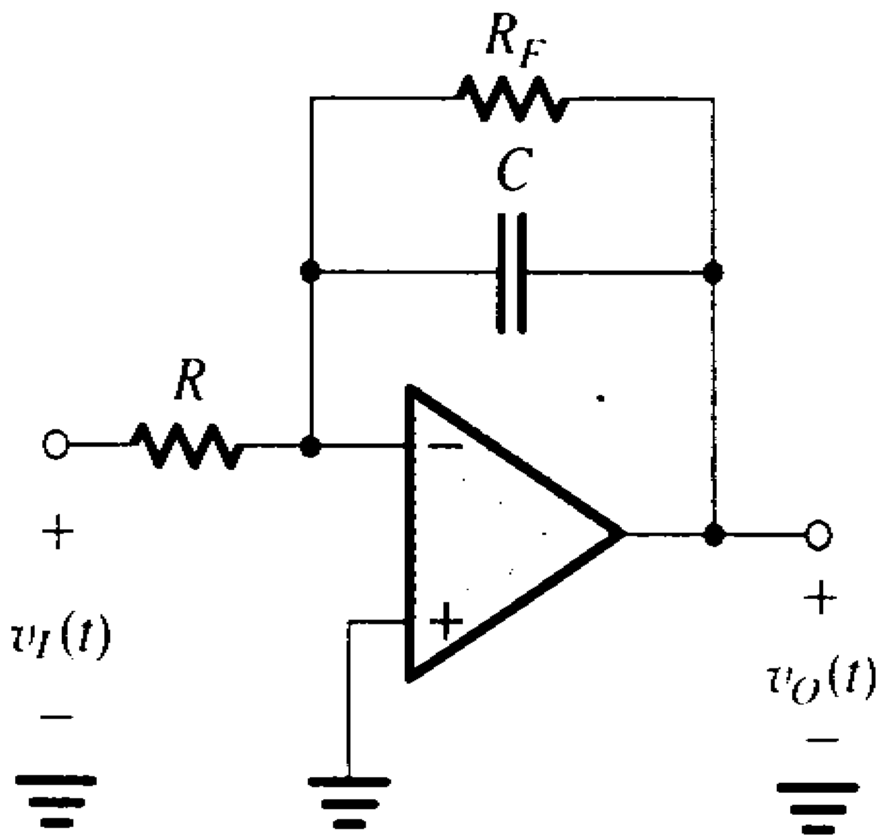
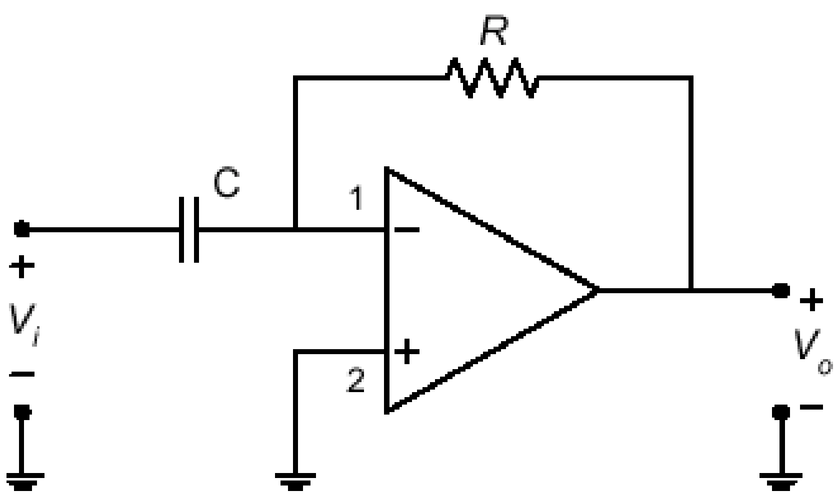
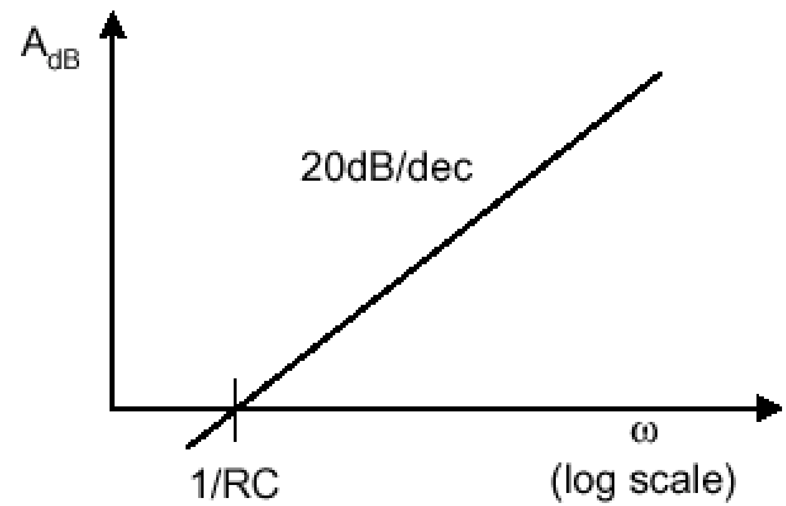


Fig. 2.12 The Miller integrator with a large resistance R_F connected in parallel with C in order to provide negative feedback at dc.

Op Amp Differentiator

The differentiator can be thought of as that of an STC high pass filter with a corner frequency at infinity.





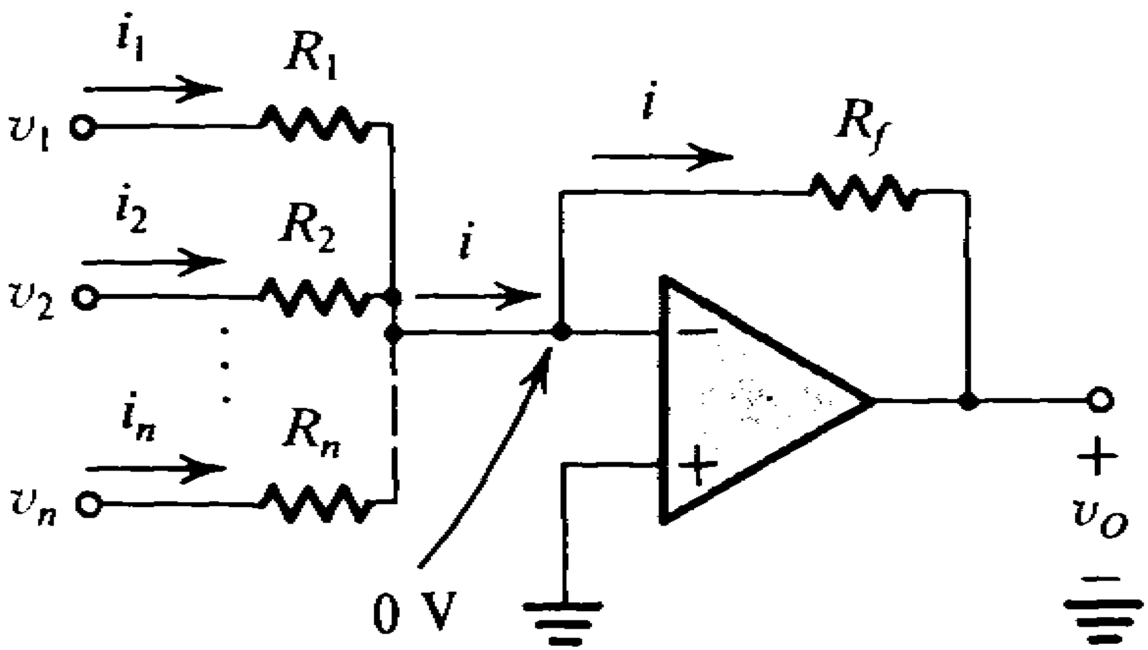
Op Amp Differentiator

$$\frac{V_O(s)}{V_I(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R}{1/sC} = -sRC$$

$$\left|\frac{V_O(j\omega)}{V_I(j\omega)}\right| = \left|-\frac{Z_2(j\omega)}{Z_1(j\omega)}\right| = \left|-j\omega RC\right| = \omega RC \;\rightarrow\; \omega = 1/RC \;\text{ when }\; A_{dB} = 0$$

- There is stability problem.
- In practice a small resistance in series with the capacitor is connected to guarantee stability.

Weighted Summer



$$i_1 = \frac{v_1}{R_1}$$

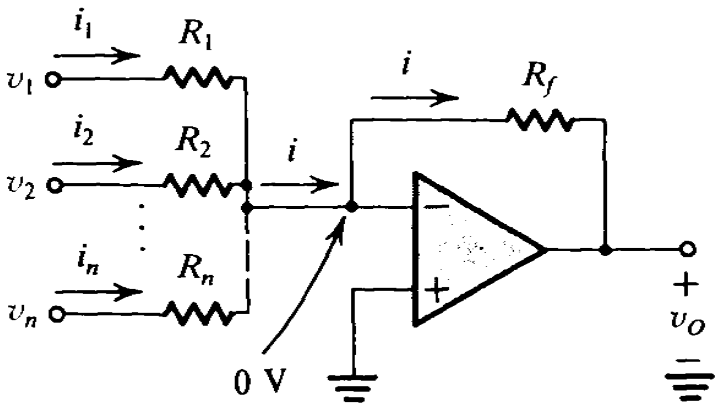
$$i_2 = \frac{v_2}{R_2}$$

$$\vdots$$

$$i_n = \frac{v_n}{R_n}$$

$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots + \frac{R_f}{R_n}v_n\right)$$

Weighted Summer



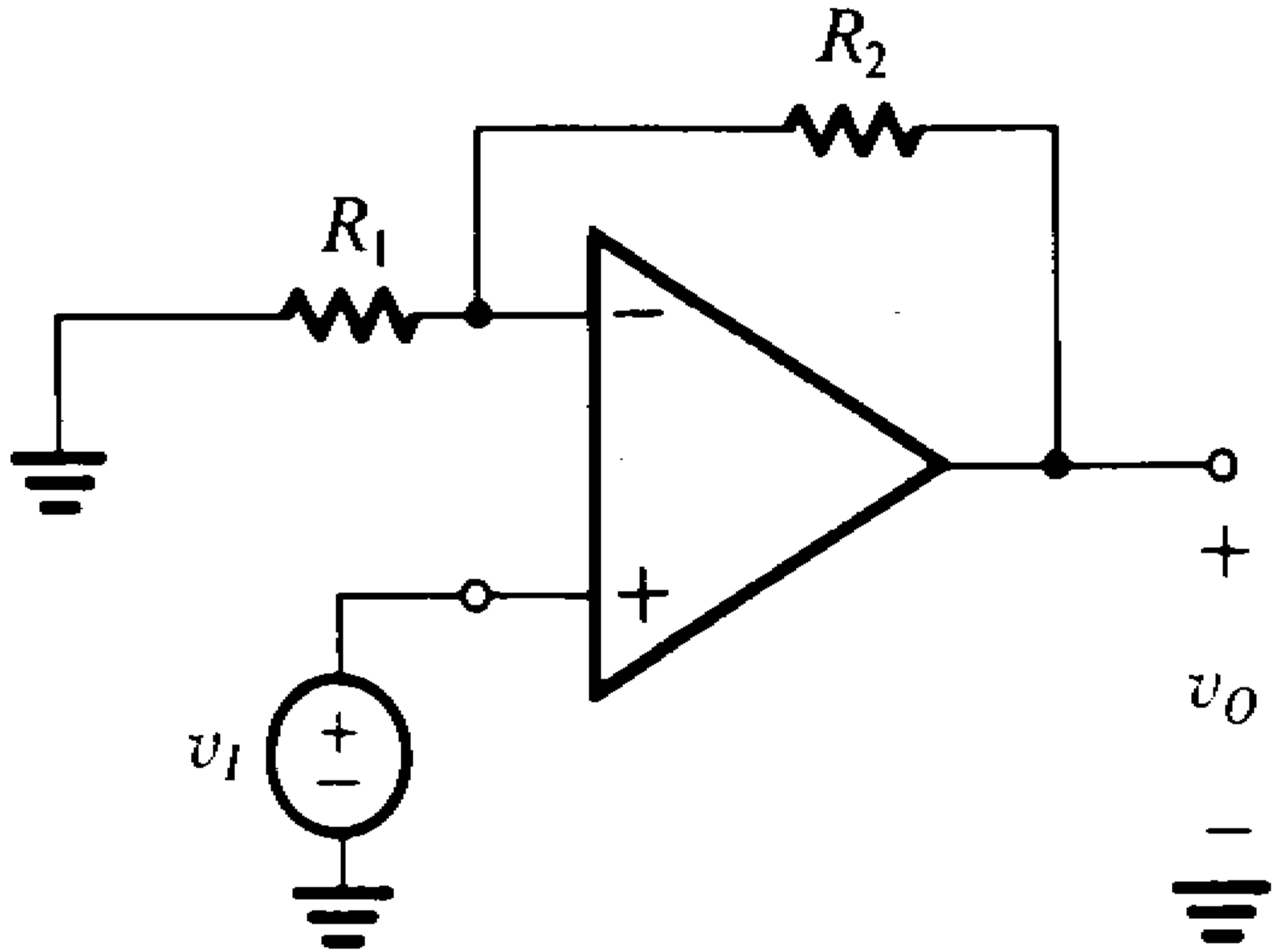
$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots + \frac{R_f}{R_n}v_n\right)$$

$$i = i_1 + i_2 + \ldots + i_n$$

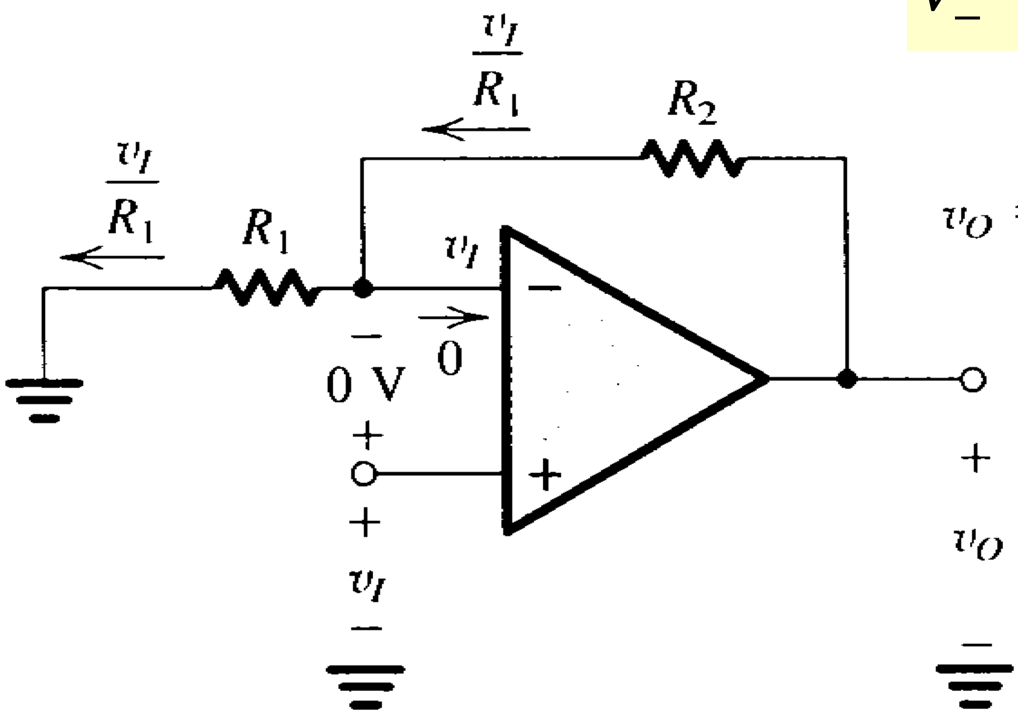
$$v_o = -iR_f = -(i_1 + i_2 + \ldots + i_n)R_f$$

$$v_o = -iR_f = -(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \ldots + \frac{R_f}{R_n}v_n)$$

Non-inverting Configuration



Noninverting Configuration



$$v_- = v_+ = v_i$$

$$v_O = v_I + \frac{v_I}{R_1}R_2 = v_I\left(1 + \frac{R_2}{R_1}\right)$$

$$i = \frac{v_-}{R_1} = \frac{v_i}{R_1}$$

$$v_0 = iR_2 + v_- = \frac{v_i}{R_1}R_2 + v_i = (1 + \frac{R_2}{R_1})v_i$$

Noninverting Configuration

When A is not infinity, we have

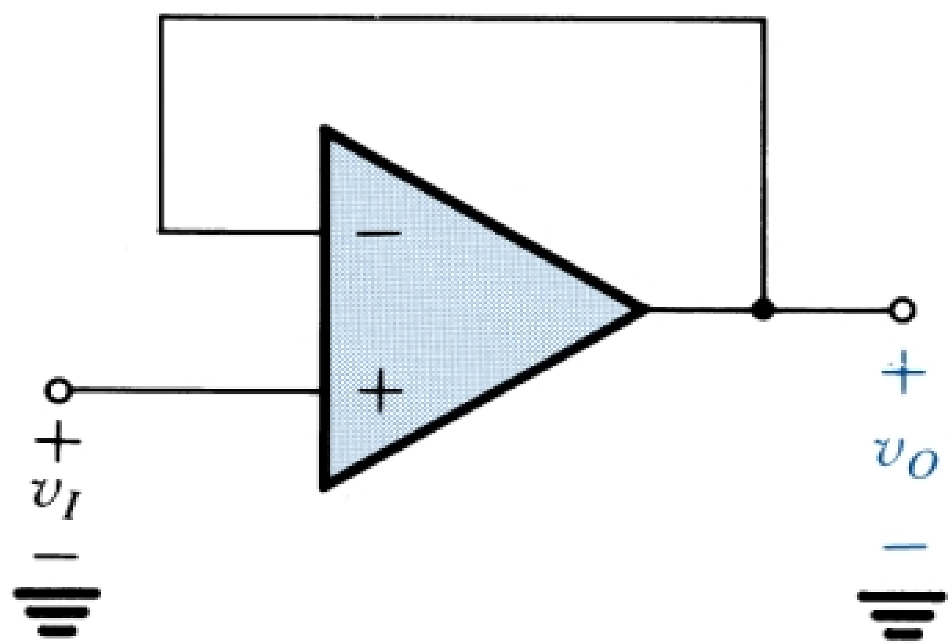
$$G = \frac{v_0}{v_i} = \frac{1 + R_2 / R_1}{1 + \dfrac{1 + R_2 / R_1}{A}}$$

Therefore the condition for an op amp to be IDEAL is:

$$1 + R_2 / R_1 << A$$

Voltage Follower

- Since the noninverting configuration has a gain greater than or equal to unity, depending on choice of R_2/R_1,
- If $R_2/R_1=0$, than the closed-loop gain is exactly 1, and the circuit is called "voltage follower"

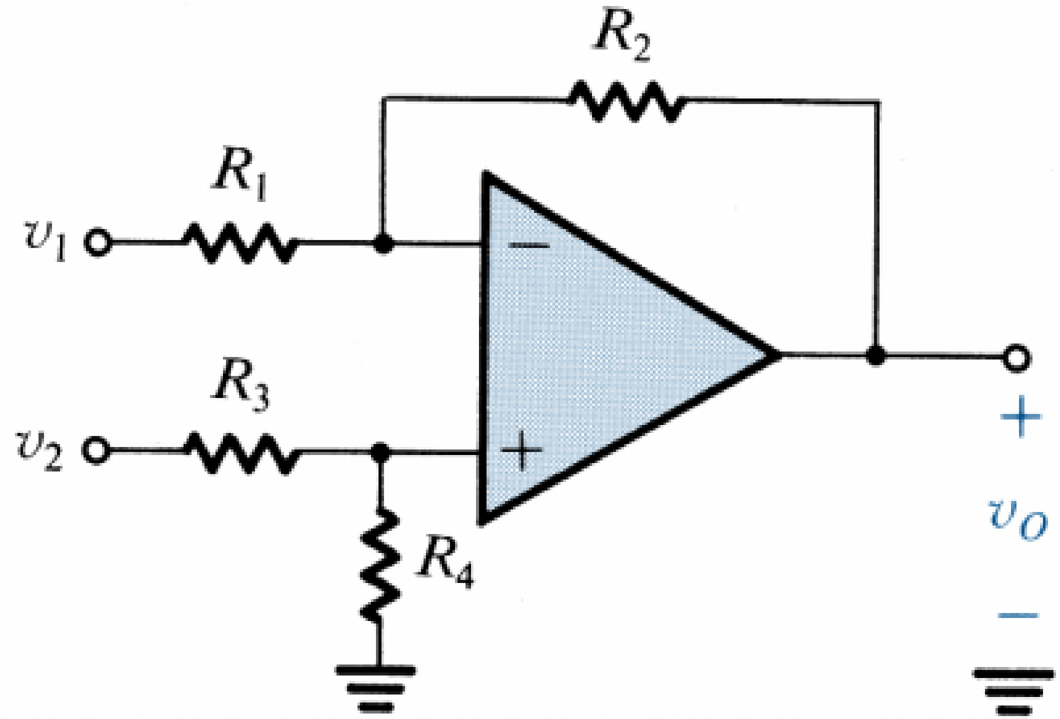


(a)

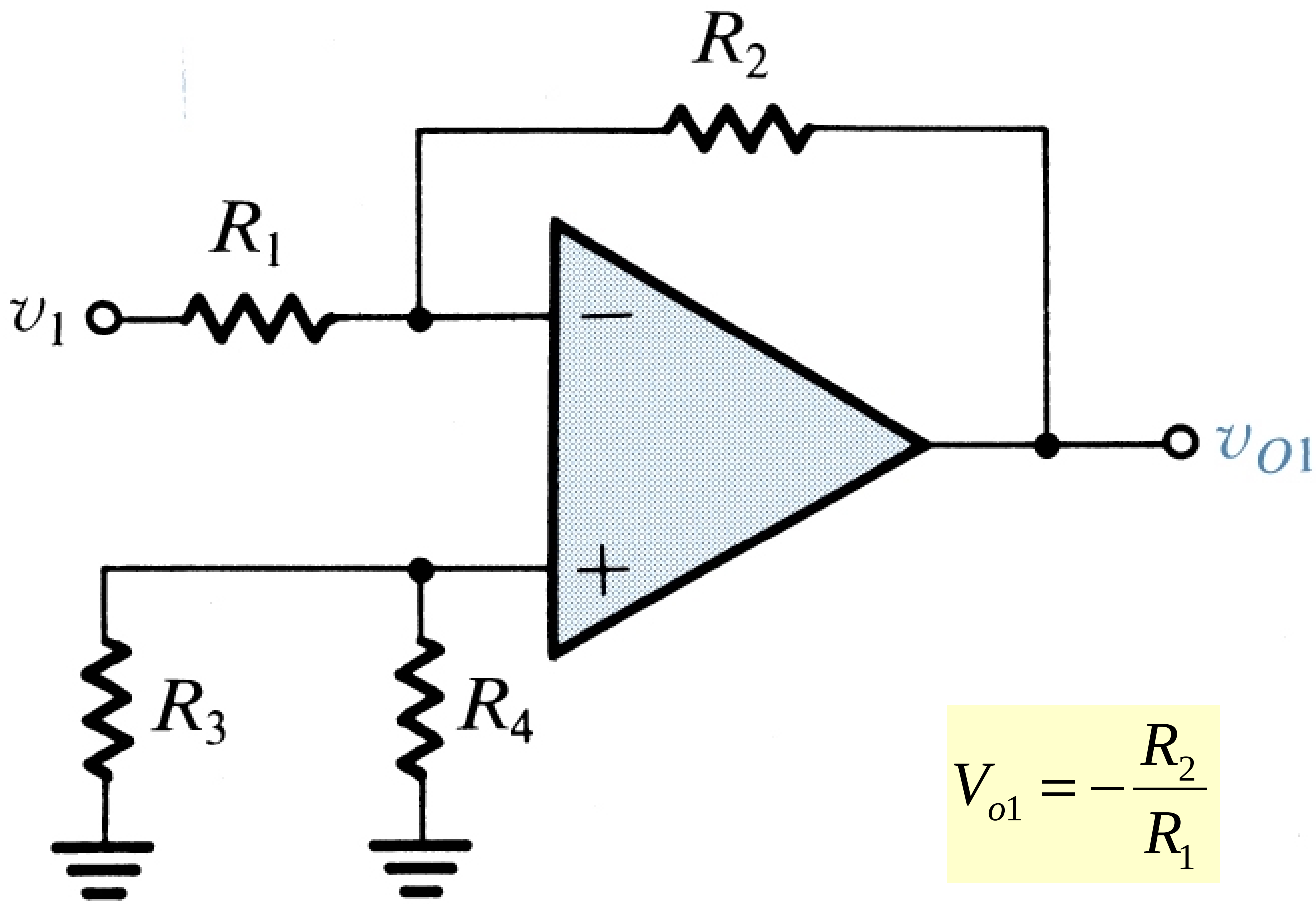
A Difference Amplifier

Now, we can combine the non-inverting amplifier and inverting amplifier configurations to be able to take a difference between two inputs. You can use superposition or brute force it…

$$\frac{v_O}{A} = v_+ - v_- \rightarrow 0 \text{ as } A \rightarrow \infty \quad \Rightarrow \quad v_+ = v_-$$



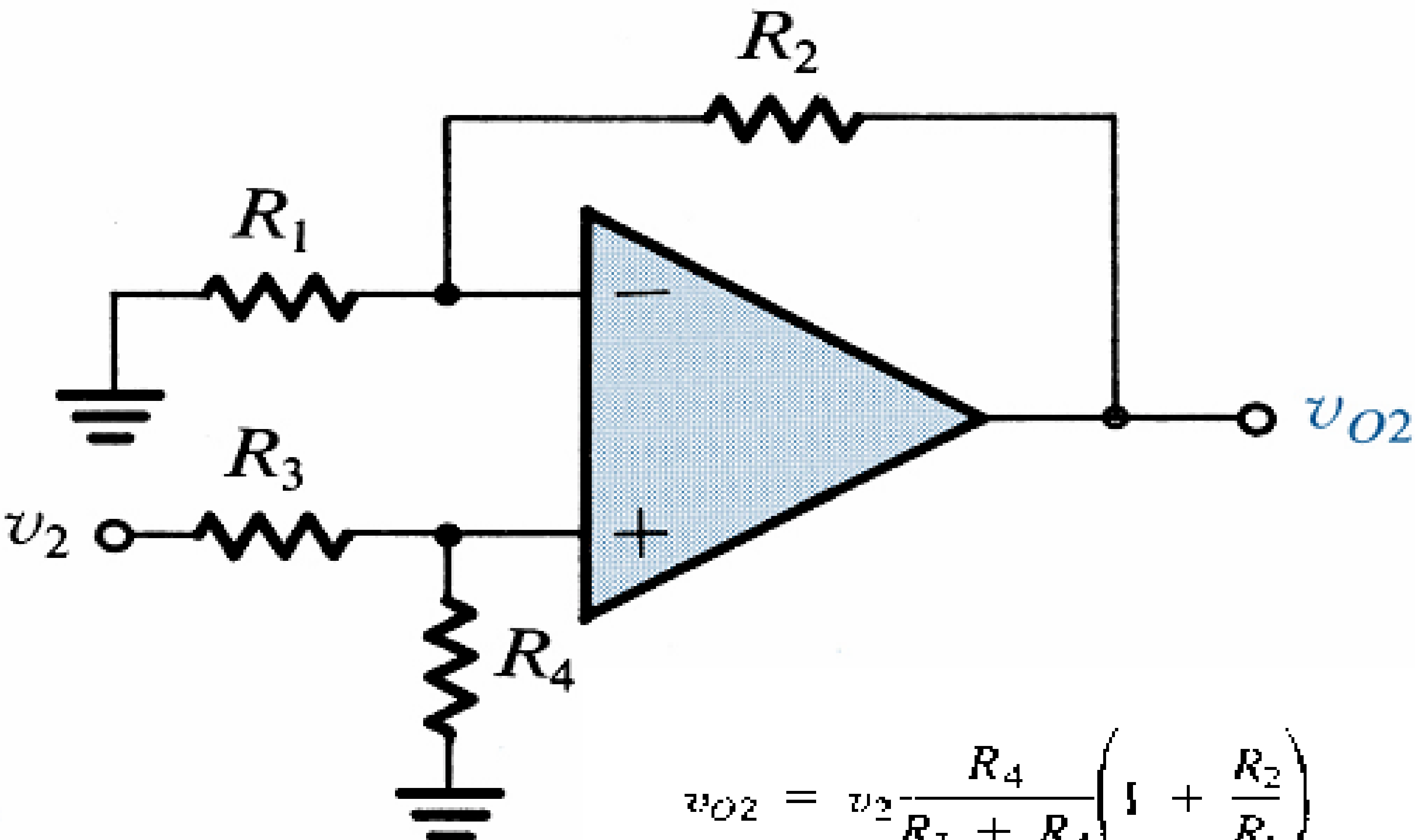
A Difference Amplifier



$$V_{o1} = -\frac{R_2}{R_1}$$

(a)

A Difference Amplifier



$$v_{O2} = v_2 \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right)$$

A Difference Amplifier

The superposition principle tells us that the output voltage v_O is equal to the sum of v_{O1} and v_{O2}. Thus we have

$$v_O = -\frac{R_2}{R_1}v_1 + \frac{1 + R_2/R_1}{1 + R_3/R_4}v_2 \tag{2.13}$$

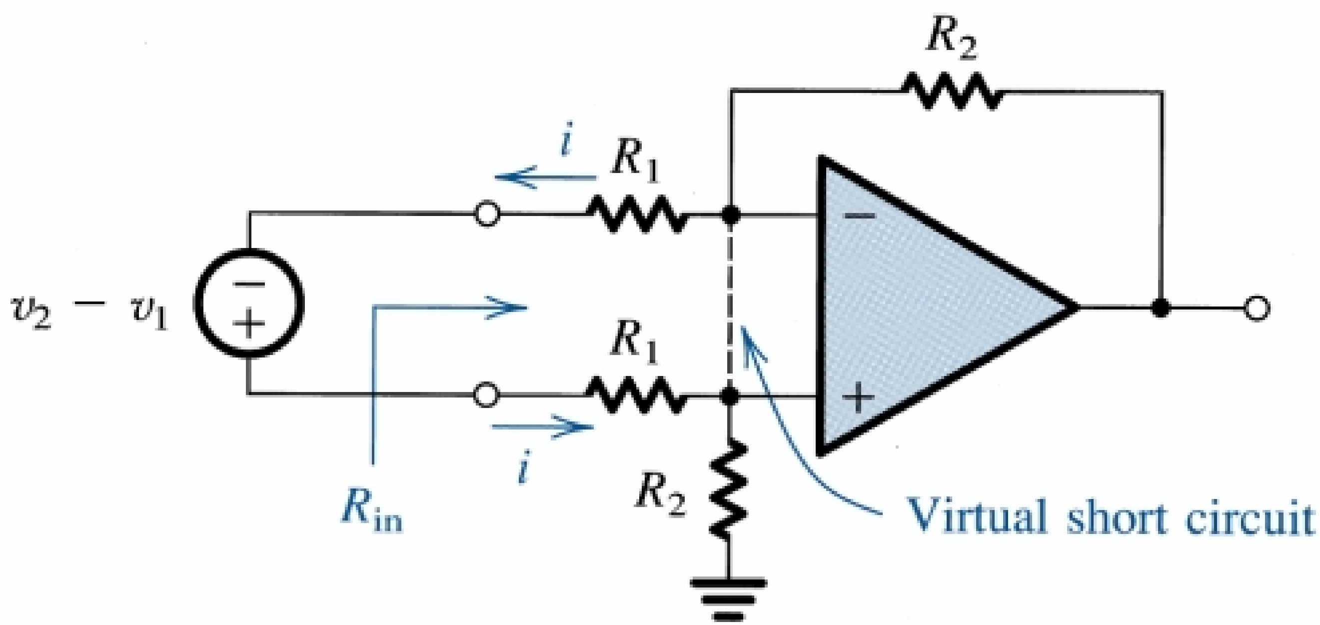
$$when \quad \frac{R_2}{R_1} = \frac{R_4}{R_3} \quad , \quad then$$

$$v_O = \frac{R_2}{R_1}(v_2 - v_1)$$

A Difference Amplifier - Input Resistance

$$R_4 = R_2, R_3 = R_1$$

$$R_{in} = \frac{v_2 - v_1}{i}$$



$$v_2 - v_1 = iR_1 + iR_1 = 2iR_1$$

$$therefore \qquad R_{in} = 2R_1$$

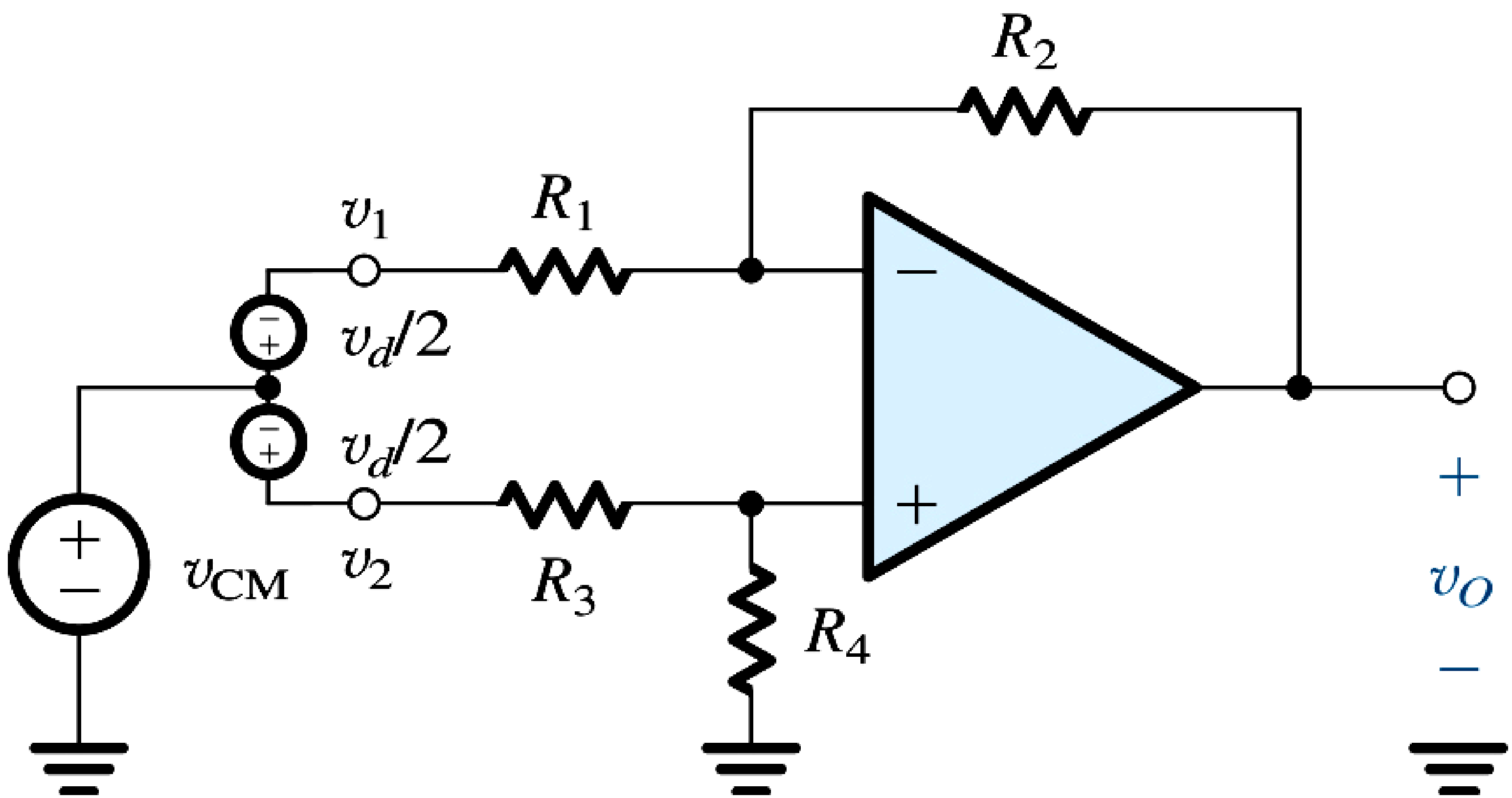
A Difference Amplifier: Why Useful?

Difference amplifiers find application in many areas, most notably an the design instrumental ion systems. As an example, consider the case of a transducer that produces between its two output terminals a relatively small signal, say 1 mV.

However, between each of the two wires (leading from the transducer to the instrumentation system) and ground there may be large picked-up interference, say 1V.

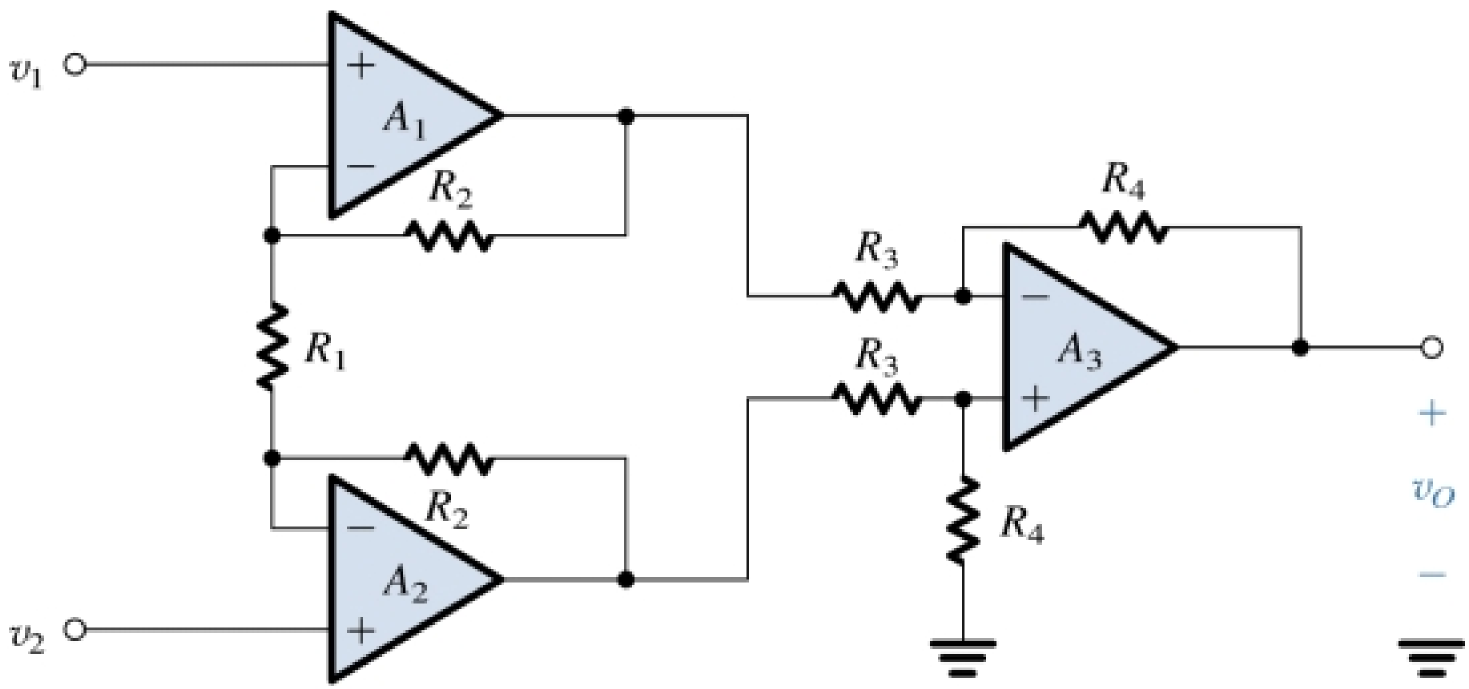
The required amplifier, known as instrumentation amplifier, must reject this large- interference signal, which is common the two wires (a common-mode signal), and amplify the small difference (or differential) signal. This situation is illustrated in next Fig. where v_{cm} denotes the common-mode signal and v_d denotes the differential signal.

A Difference Amplifier



An Instrumentation Amplifier

Problem with the circuit as an instrumentation amplifier is the low resistance. Therefore the following circuit was proposed:



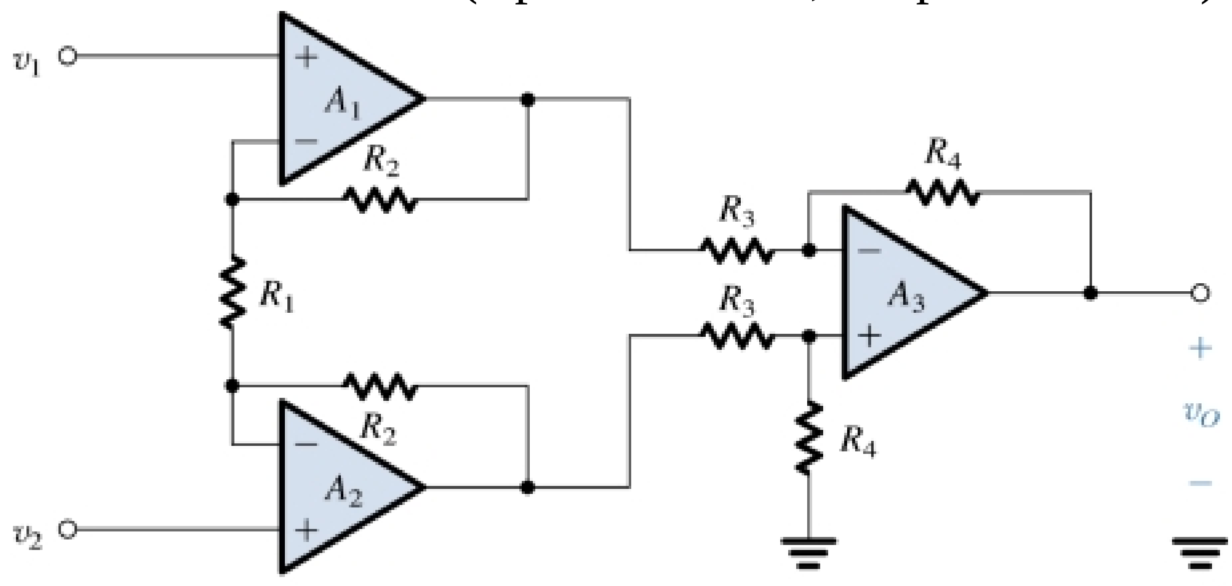
(a)

An Instrumentation Amplifier

The difference amplifier studied in the previous example is not entirely satisfactory as art instrumentation amplifier its major drawbacks are its low input resistance and that its gain cannot be easily varied.

A much superior instrumentation amplifier circuit. Analyze the circuit to determine v_o as a function of v_1 and v_2, and determine the differential gain. Suggest a way for making the gain variable. Also find the input resistance.

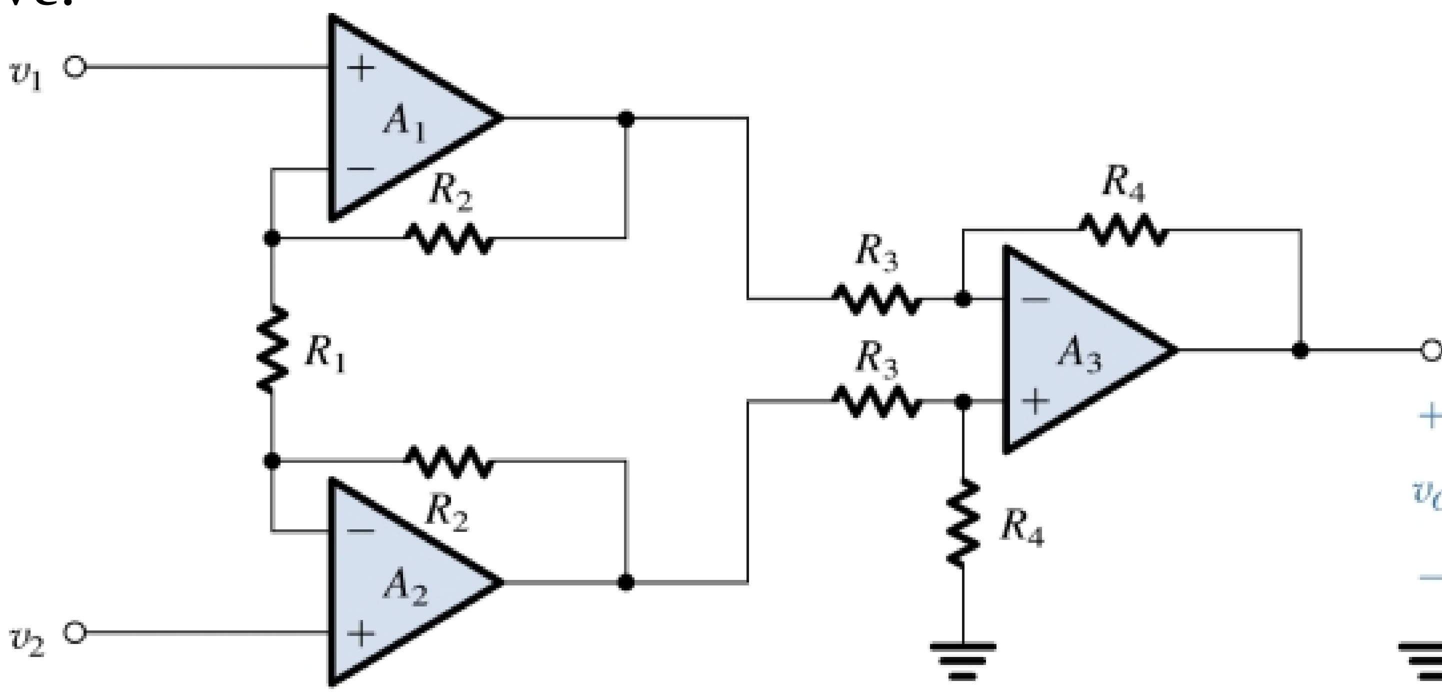
Design the circuit to provide a gain that can be varied over the range 2 to 1000 utilizing a 100-kΩ variable resistance (a potentiometer, or "pot" for short)



(a)

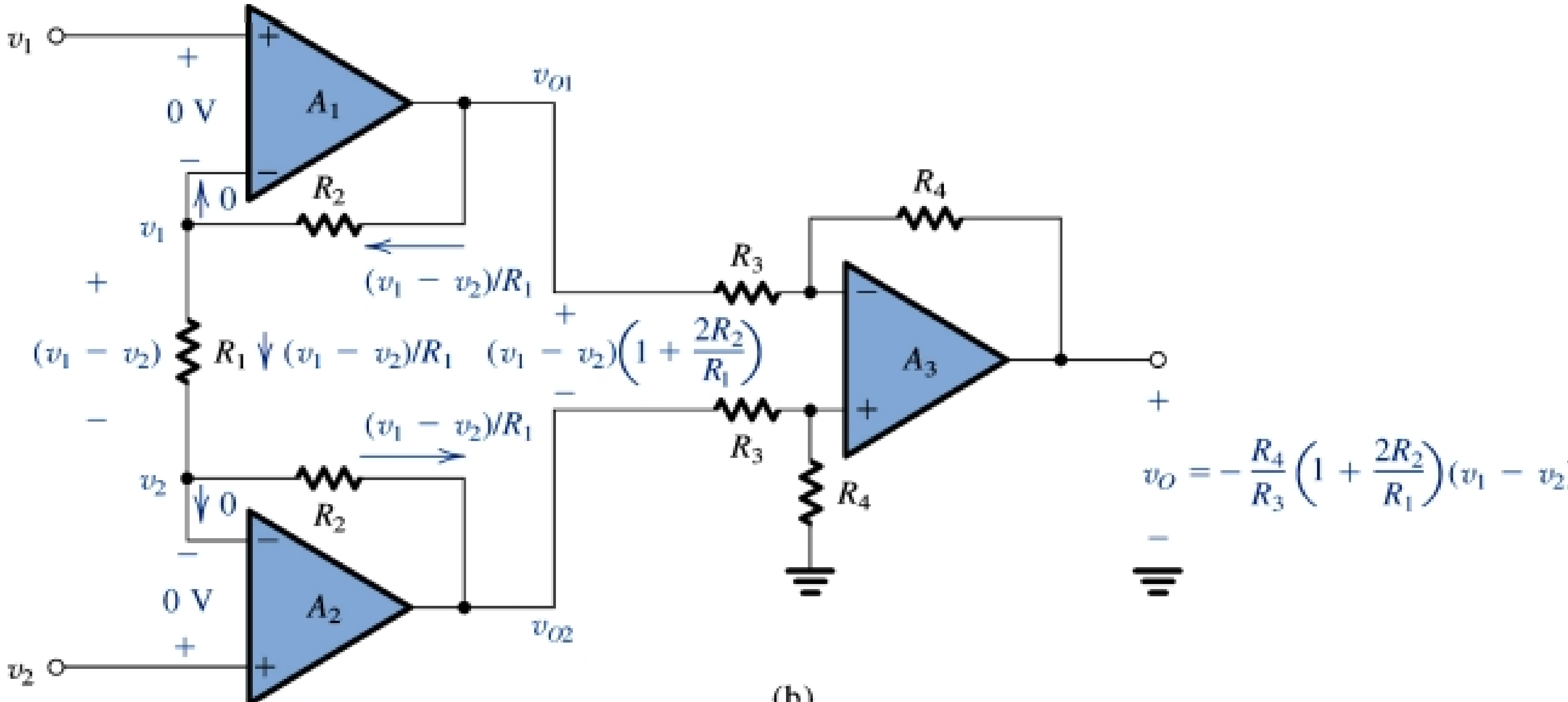
An Instrumentation Amplifier

The circuit consist of two stages; The first stage is formed by op amps A_1 and A_2: and their associated resistors, and the second stage is formed by op amp A_3 . Together with its four associated resistors. We recognize the second stage as that of the difference amplifier studied above.



(a)

An Instrumentation Amplifier



(b)

$$v_{O1} - v_{O2} = \left(1 + \frac{2R_2}{R_1}\right)(v_1 - v_2)$$

An Instrumentation Amplifier

The difference amplifier formed around op amp A_3 senses the voltage difference $(v_{O1} - v_{O2})$ and provides a proportional output voltage v_O,

$$v_O = -\frac{R_4}{R_3}(v_{O1} - v_{O2}) \quad (2.15)$$

Combining Eqs. (2.14) and (2.15) results in

$$v_O = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)(v_2 - v_1)$$

Thus the instrumentation amplifier has a differential voltage gain

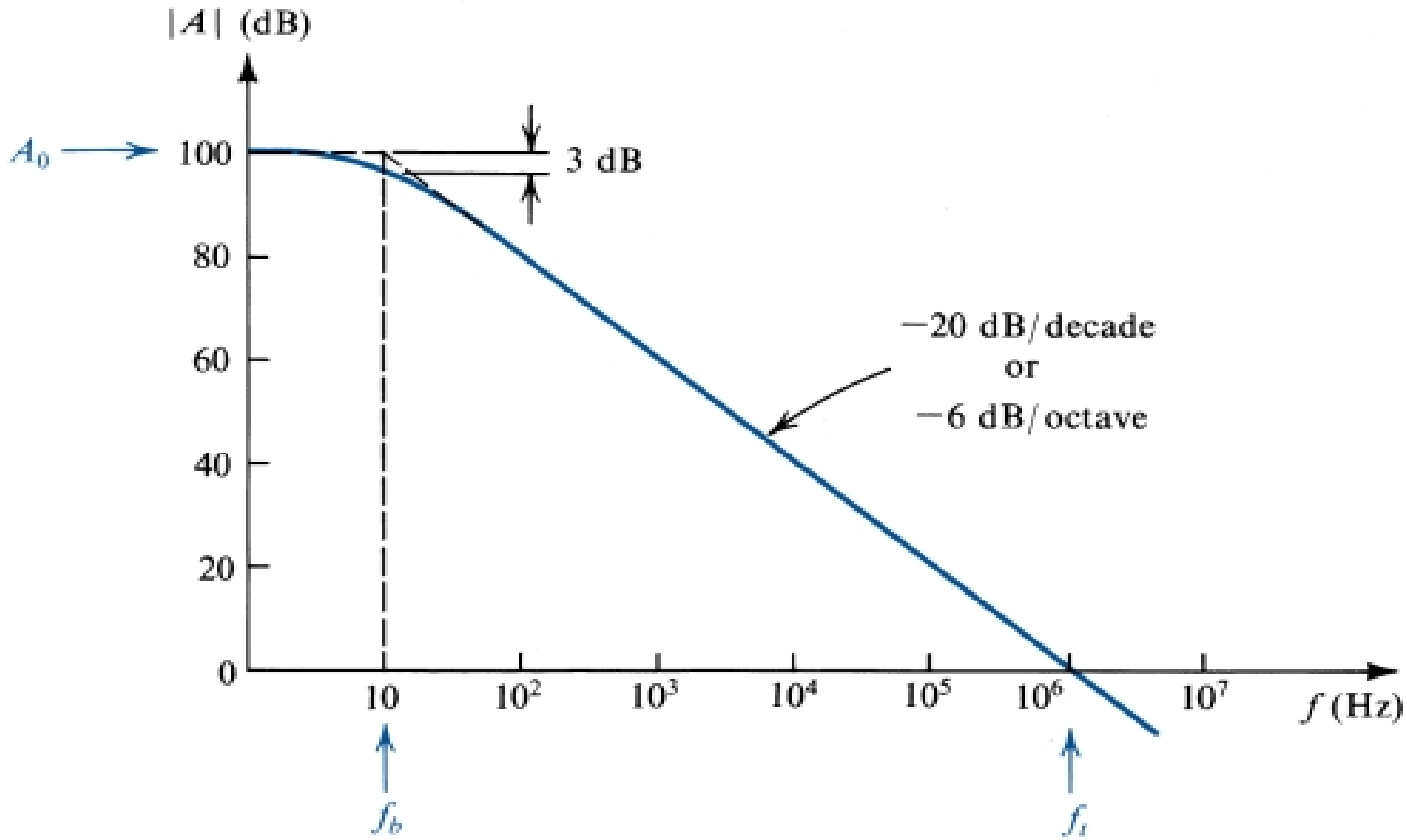
$$A_d \equiv \frac{v_O}{v_2 - v_1} = \left(1 + \frac{2R_2}{R_1}\right)\frac{R_4}{R_3} \quad (2.16)$$

An Instrumentation Amplifier

- From the differential-gain expression in Eq. (2-16) we observe that the gain value can be varied by varying the single resistor R_1 any other arrangement involves varying two resistors simultaneously.
- Since both of the input-stage op amps are connected in the noninverting configuration.
- The input impedance seen by each of v_1 and v_2 is (ideally) infinite. This is a major advantage of this instrumentation amplifier configuration.

Effect of Non-ideal Open-Loop Gain

The differential open-loop gain of an op amp decreases with frequency



Effect of Non-ideal Open-Loop Gain

- Note that although the gain is quite high at dc and low frequencies, it starts to fall off at a rather low frequency (10 Hz in our example).
- The uniform -20-dB/decade gain roll-off shown is typical of internally compensated op amps. These are units that have a network (usually a single capacitor) included on the same IC chip whose function is to cause the op-amp gain to have the single-time-constant low-pass response shown.
- This process of modifying the open-loop gain is termed frequency compensation, and its purpose is to ensure that op-amp circuits will be stable (as opposed to oscillatory).

Effect of Non-ideal Open-Loop Gain

By analogy to the response of low-pass STC circuits (see Section 1.6 and, for more detail, Appendix F), the gain $A(s)$ of an internally compensated op amp may be expressed as

$$A(s) = \frac{A_0}{1 + s/\omega_b} \tag{2.17}$$

which for physical frequencies, $s = j\omega$, becomes

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b} \tag{2.18}$$

Effect of Non-ideal Open-Loop Gain

3-dB frequency ("break" frequency) : $$\left|A(\omega_b)\right| = \frac{1}{2}\left|A_0\right|$$

Unit-Gain frequency (or unit-gain bandwidth) the frequency at which the gain is 1: for $\omega >> \omega_b$

$$\left|A(\omega)\right| \approx \left|A_0 \frac{\omega_b}{\omega}\right|$$

Hence Unit-Gain frequency is $$\omega_t \approx A_0 \omega_b$$

The open-loop gain of an op amp can be estimated using

$$\left|A(\omega)\right| \approx \left|A_0 \frac{\omega_b}{\omega}\right| = \frac{\omega_t}{\omega}$$

The unit-gain bandwidth is usually given by data sheet

Frequency Response of Closed-Loop Amplifiers

- The closed-loop gain for inverting amplifier, assuming a finite opamp open-loop gain A, was already derived as:

$$\frac{V_o(s)}{V_i(s)} = \frac{-R_2/R_1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}\right)}$$

$$A = \frac{A_0}{1+\frac{s}{\omega_b}}$$

- Substituting A gives:

$$\frac{V_o(s)}{V_i(s)} = \frac{-R_2/R_1}{1+\frac{1}{A_0}\left(1+\frac{R_2}{R_1}\right)+\frac{s}{\omega_t/(1+R_2/R_1)}}$$

- For $A_0 >> 1 + R_2 / R_1$ which is usually the case

$$\frac{V_o(s)}{V_i(s)} = \frac{-R_2/R_1}{1 + \dfrac{s}{\omega_t/(1+R_2/R_1)}}$$

- which is of the same form as that for a low-pass single-lime-constant network), ω_t
- Thus the inverting amplifier has an STC low-pass response with a dc gain of magnitude equal to *R2/R1*.
- The closed-loop gain rolls off at a uniform —20 dB/decade slope with a comer frequency (3-dB frequency) given by

$$\omega_{3dB} = \frac{\omega_t}{1 + R_2/R_1}$$

Frequency Response of Closed-Loop Amplifiers

Similarly, analysis of the noninverting amplifier of Fig. 2.16, assuming a finite open-loop gain A, yields the closed-loop transfer function

$$\frac{V_o}{V_i} = \frac{1 + R_2/R_1}{1 + (1 + R_2/R_1)/A} \tag{2.28}$$

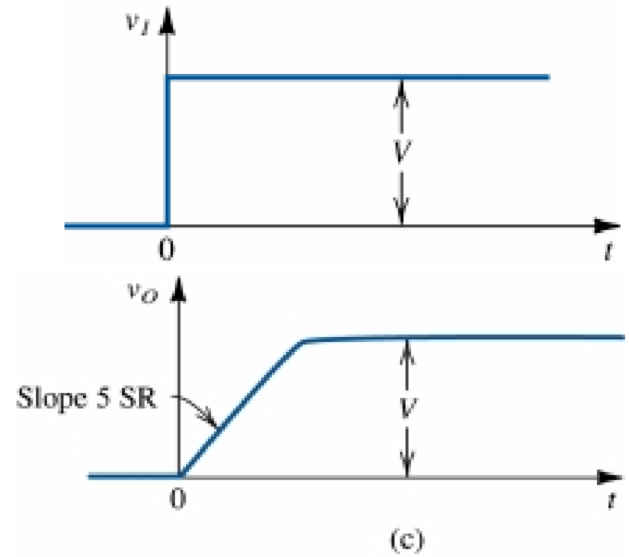
Substituting for A from Eq. (2.17) and making the approximation $A_0 \gg 1 + R_2/R_1$ results in

$$\frac{V_o(s)}{V_i(s)} \simeq \frac{1 + R_2/R_1}{1 + \dfrac{s}{\omega_t/(1 + R_2/R_1)}} \tag{2.29}$$

Thus the noninverting amplifier has an STC low-pass response with a dc gain of $(1 + R_2/R_1)$ and a 3-dB frequency given also by Eq. (2.27).

Large-Signal Operation of Op-Amps

- input step waveform
- Slew-rate limited
- Exponential rising output



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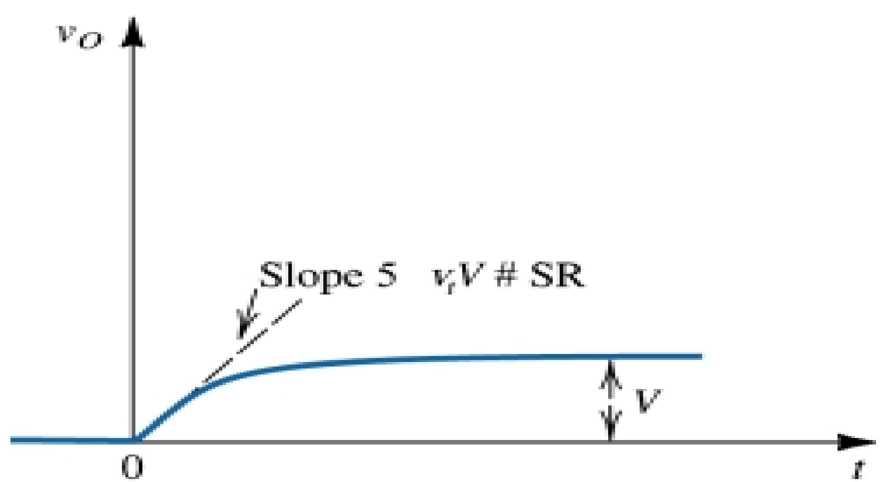

(d)