**DT287/3**

*Transmission line questions*

**Q1**

Measurements on a 50 Ω transmission line produced a maximum voltage $V_{\text{max}} = 10$ mV and a minimum voltage $V_{\text{min}} = 2$ mV at an operating frequency $f = 1$ GHz. The first voltage minimum from the load was measured at 25.5 mm. If $\varepsilon_r = 2$, determine

(i) The VSWR,

(ii) The wavelength,

(iii) The magnitude of the reflection coefficient, and

(iv) The distance in wavelengths between the load and the first minimum.

[8 marks]

It is required to match the load to the line using a short-circuited single stub having the same characteristics as the line. Determine the location and length of the stub for correct matching.

[12 marks]

**Q2**

(a) A 10 km length of screened telephone cable, operating at a frequency of 10 kHz, has the following primary transmission line parameters:

- $L = 700$ mH per km,
- $C = 0.05$ µF per km,
- $R = 28$ Ω per km, and
- $G = 1$ µS per km

Determine the phase and attenuation constants for this cable and hence calculate the characteristic impedance.

[8 marks]

(b) An expression for the input impedance of a transmission line, terminated in a resistance $Z_R = 35$ Ω is:

$$Z_{in} = Z_o \left[ \frac{Z_R + jZ_o \tan \beta \ell}{Z_o + jZ_R \tan \beta \ell} \right]$$

Where $\beta$ is the phase change coefficient and $Z_o$ is the characteristic impedance equal to 75 Ω. Determine the input impedance of a section of line whose length is $\lambda/4$ metres.

[7 marks]

(c) Describe one graphical technique for matching a transmission line to a load, whose value is not equal to the characteristic impedance. Illustrate your answer using a Smith chart.

[10 marks]

**Q3**

(a) A 1 km length of screened telephone cable, operating at a frequency of 10 kHz, has the following parameters:

- $L = 700$ mH per km,
- $C = 0.05$ µF per km,
\[ R = 28 \, \Omega \text{ per km, and} \]
\[ G = 1 \, \mu\text{S per km} \]

Determine the characteristic impedance, phase and attenuation constants for this cable.

(b) A transmission cable is terminated by an impedance which has a value twice the characteristic impedance value. Calculate the position and length of a short-circuited stub required to achieve matched conditions on the line. The stub is constructed from the same cable type as the line and the frequency of operation is 1 MHz. The velocity of propagation is 0.67c, where c is the velocity of light.

If this cable is terminated by an impedance \( Z_L = 123.69 - j36.01 \, \Omega \), calculate the position and length of a short-circuited stub required to achieve matched conditions on the line. The stub is constructed from the same cable type as the line.

\[ \text{Q4} \]

(a) Define the term voltage reflection coefficient. Use the solution for the voltage along a transmission line \( V = V_1 e^{\alpha z} + V_2 e^{\beta z} \) in your answer.

(b) A transmission line with a 75 \, \Omega \text{ characteristic impedance } \( Z_0 \), is terminated in an impedance \( Z_L = 40 + j20 \, \Omega \). Calculate

(i) The reflection coefficient,
(ii) The voltage standing ratio, and
(iii) The input impedance.

(c) Hence use a Smith chart to verify approximately, the three values calculated in (i) to (iii).

\[ \text{Q5} \]

(a) Explain the transmission line terms: Voltage standing wave ratio and voltage reflection coefficient.

(b) A twin-lead transmission line whose characteristic impedance \( Z_0 = 300 + j0 \, \Omega \) has an aerial of impedance 225 - j175 \, \Omega \text{ connected as a load. Matching by means of a single stub connected a distance d metres from the load is used. Estimate the length in metres of the stub and the distance d if the operating frequency } f = 500 \text{ MHz assume that the stub is formed from a section of the same air-spaced transmission line.}
iv) Going around the constant VSWR circle to the point where the intersection with the constant conductance circle of 1 (i.e. through the circle $G/Y = 1.0 + j0.72$).

v) A stub is attached whose admittance $y_s = -j0.72$ will cancel out the admittance $j0.72$ s. The points M and B, which are read from the chart as $M = 0.131 \lambda$ towards the generator and $B = 0.153 \lambda$. The difference between these two measurements is $d = (0.153 - 0.131) \lambda = 0.022 \lambda$ metres towards the generator. The relationship $v = \lambda/f$ gives us a way of calculating the distance $d$ in metres i.e. $\lambda = v/f = 300.108/500.108 = 0.6m$ or 60 cm so that the distance $d = 0.022 \times 60 \text{ cm} = 1.32 \text{ cm}$

vi) The length of the stub is found from the distance $= (0.25-0.099) \lambda = 0.151 \lambda$ so that the length of the stub is $= 0.151 \times 60 \text{ cm} = 9.06 \text{ cm}$

Q6

A transmission line whose characteristic impedance $Z_o$ is $(75 + j0) \Omega$, has a velocity factor of 0.6 and negligible attenuation. Determine, using a Smith chart, the VSWR on the line given that the transmission line is terminated in an impedance $Z_R = 187.5 + j187.5 \Omega$ and the operating frequency is $f = 90 \text{ MHz}$.

On the chart, determine the length and location of a single stub which will achieve correct matching. You may assume the stub is made from a line with the same transmission characteristics.

Determine, with the aid of a Smith chart, the characteristic impedance of a quarter wave transmission line to achieve correct matching. You may assume the transformer is inserted at a point nearest the load.

Q7

a) A load $Z = 100 - j50 \Omega$ is connected to a transmission line whose characteristic impedance is $75 \Omega$. Determine, with the aid of a Smith chart, the characteristic
impedance of a quarter wave transmission line to achieve correct matching. You may assume the transformer is inserted at a point nearest the load. [8 marks]

b) A transmission line with characteristic impedance $Z = 300 + j0 \, \Omega$, is connected to aerial of impedance $Z_L = 225 - j175 \, \Omega$ A single -stub is connected a distance $d$ metres from the load. If the frequency of operation is $500 \, \text{MHz}$, estimate the length of stub required for correct matching. (You may assume the stub is formed from the same section of transmission line and the velocity of propagation is $2.01 \times 10^8 \, \text{m/s}$).

[12 marks]

Q8

A transmission line whose characteristic impedance is $75 + j0 \, \Omega$, a velocity factor of $0.6$ (Where $c$ is the velocity of light) and negligible attenuation is used to connect a transmitter to a load .The operating frequency of the transmitter is $90 \, \text{MHz}$ and the impedance of the load is $187.5 + j187.5 \, \Omega$. Determine using a Smith chart

a) The VSWR on the line, [2marks]

b) the physical length and position of a single short-circuited stub which will achieve correct matching. Two solutions are possible. State with reasons the preferred solution. [12marks]

c) If the transmitter frequency is reduced by 10 %, determine using a separate Smith chart, the VSWR for the solution in b) [6marks]

Solution

The line wavelength is calculated as :

wavelength $\lambda = 2 \, \text{m}$

The normalised impedance is $Z/Z_i$ i.e. $2.5 + j2.5$. This is located at point A on the chart. The normalised admittance is $0.2 + j0.2$ and is at point B.

a) Position of stub relative to the load :

$d (l) = 0.0033 + 0.184 = 0.217l$

so that the length is $0.217 \times 2 \, \text{metres} = 0.434 \, \text{m}$

The normalised input of the line at the stub position is

$y = 1 + j1.85$

The normalised suscpeptance of the stub is $s1$. The required length of the stub $s1$

$d (l) = (0.033 + 0.316)l = 0.349 \times 2 \, \text{metres}$

The normalised input admittance of the line at the stub position :

The first solution is preferable since the stub is shorter and is positioned closer to the load. For a frequency change of - 10 % $f = 0.9 \, \text{MHz}$ so that the normalised impedance becomes $2.5 + j2.25$ see point A chart 2 and $y' = 0.23 + j0.2$ see point B chart 2. Electrical length of the line between the load and the stub is :
\[ d' = 0.217 \times 0.9 = 0.195\lambda \]

Normalised input admittance of the line at the stub position:

\[ y' = 0.675 + j\ 1.36 \text{ see point C chart 2} \]

Electrical length of the stub for the changed frequency is

\[ l' = 0.785 \times 0.9 = 0.07\lambda \]

Normalised susceptance of the line:

\[ y' = -j\ 2.15 \text{ see point F on chart 2} \]

Normalised admittance of line and stub at stub position

\[ y' = y' + y' = 0.675 + j\ 1.36 - j\ 2.15 \]

The resulting VSWR on the line is given by the intersection at E

\[ VSWR = 2.8 \]

The VSWR of the unmatched line is 5

Q9

**Explanation of single stub method**

A 15 km length of loss-free transmission line is terminated by a 400 \( \Omega \) resistor. A leakage fault develops across the transmission line at a point \( p \) km from the sending end. To locate the fault the method of TDR is used. A record of the transmitted pulse (sent at \( t = 0 \text{ s} \)) and the first two reflections is shown in Fig.1. Calculate

a) The inductance and the capacitance of the line per unit length,
b) The position of the fault and the magnitude of its conductance.

Q9

[6 marks]
In a loss-free line the velocity of propagation is $3 \times 10^6$ m/s so in 20 us a pulse travels a distance of 6 km. Hence the fault is 3 km down the line and the -3 V is the reflected voltage from the fault point.

Solution

Consider the Bewley diagram below.

At the fault let the total impedance $Z$ be the parallel combination of the fault resistance $R$ and the characteristic impedance. We can then write

$$Z = \frac{RZ_0}{R + Z_0}$$

The reflection coefficient is defined as the ratio of the incident voltage to the reflected voltage. So we can then write

$$\rho = \frac{RZ_0}{R + Z_0}$$

The transmitted voltage beyond $P$ is calculated by considering the fundamental relationship for the total voltage $V$ at a point on line where due to a mismatch there is a reflection so But the reflection coefficient is given as where $Z$ is the terminating impedance at the point in question. Substituting c) into b) yields The transmitted beyond $P$ is This wave travels to $Q$ where it causes an unknown reflection $x$ volts back towards the source, and beyond $P$ we can write and substituting from a) solving for $R$ or a conductance of For a loss-free line we can write The velocity of propagation $V = \frac{1}{2\pi}$ From these two equations we can write for $C$ as and hence the inductance $L$ is calculated as $L = 379,456 \times 5.41 \times 10 = 2,051 \mu$H per metre

**Q10**

(a) Standing waves were observed on a 50 $\Omega$ transmission line with a maximum voltage $V_{\text{max}} = 10$ mV. The minimum voltage $V_{\text{min}} = 2$ mV was measured 50.8 mm from the load. If the wavelength for the operating frequency is 212 mm, determine:

(i) The VSWR,

(ii) The magnitude of the reflection coefficient $\rho$,

(iii) The distance in wavelengths between the load and the first minimum, and
(iv) The mismatch loss in dB. [8 marks]

(b) Determine the location and the length of a single stub (same characteristics as the mismatched line) for correct matching. [10 marks]

(c) Describe the technique of TDR for measuring faults on transmission lines. [7 marks]

**Q11**

(a) Define the following transmission line terms: Voltage standing wave ratio and voltage reflection coefficient. [5 marks]

(b) A transmission line, whose characteristic impedance \( Z_0 = 300 + j0 \) \( \Omega \), has an antenna of impedance \( 225 - j175 \) \( \Omega \) connected as a load. Matching by means of a single stub connected, at a distance \( d \) metres from the load, is used. Estimate the length in metres of the stub and the distance \( d \) if the operating frequency \( f = 500 \) MHz. Assume that the stub is formed from a section of the same air-spaced transmission line. [12 marks]

(c) Calculate the characteristic impedance, phase coefficient and attenuation constant for a 10 km length of telephone cable, operating at a frequency of 10 kHz, if the cable has the following, primary transmission line parameters:
\[
L = 700 \, \text{mH per km}, \quad C = 0.05 \, \mu\text{F per km}, \quad R = 28 \, \Omega \, \text{per km}, \quad G = 1 \, \mu\text{S per km}
\]

**Q12**

A 50 \( \Omega \) transmission line is connected to an antennae with \( Z_{\text{ant}} = 100 - j60 \) \( \Omega \). Determine the following for a single stub tuner:

a) Shortest distance to the stub (from the antennae load),
b) Value and type of reactance needed to tune(resonant) the line, and
c) Length of short-circuited stub.

**Solution**

a) 0.126\( \lambda \),
b) Inductive reactance = \( j44.25 \) \( \Omega \), and
0.115\( \lambda \)

**Q13**

A load of \( Z_{\text{ant}} = 100 - j60 \) \( \Omega \) is to be matched to a 50 \( \Omega \) line with a quarter wave transformer.

(a) Where should the transformer be located,
(b) Determine the characteristic impedance of the transformer, and
(c) If \( f = 1 \) GHz, determine the length of the transformer in metres if the transmission line dielectric constant, is \( e = 2.25 \) S

**Solution**

0.21\( \lambda \) from the load
(b)29.6 \( \Omega \), and
(c)0.05\( \lambda \)
Active Filters Questions

Q14

(a)
Obtain a transfer function for a high-pass filter, which uses a Butterworth loss function, and which meets the following specification:
The maximum passband loss $A_{\text{max}} = 3 \text{ dB}$
The minimum stopband loss $A_{\text{min}} = 28 \text{ dB}$
The passband edge frequency $\omega_p = 6000 \text{ rad/s}$
The stopband edge frequency $\omega_s = 2000 \text{ rad/s}$

(b) Show how the transfer function, for the second stage of the Sallen and Key, high-pass filter shown in figure 4, is:

$$\frac{E_1}{E_2} = \frac{s^2}{s^2 + s \frac{2}{CR_s} + \frac{1}{C^2 R_s R_2}}$$

Hence calculate component values for this circuit, which could implement the transfer function obtained in part (a).

$R = 20 \text{ k}\Omega$.

Q15

(a) Discuss the effect, component tolerance, has on the performance of active filters.

(b) The frequency spectrum of a 1 kHz squarewave is shown in figure 5. It is desired to extract, from this squarewave, a 1 kHz sinusoidal signal. Obtain a transfer function for a low-pass filter, which will produce a maximum attenuation of the fundamental component of 1 dB. The third harmonic (3 kHz) should be attenuated by 12 dB (Butterworth loss functions tables are available for use in your analysis).
Q16

(a) It is desired to produce a 1 kHz sinusoidal signal from a 1 kHz squarewave. Obtain the transfer function for a low-pass filter that will produce a maximum attenuation of the fundamental signal by 1 dB. The third harmonic should be attenuated by 12 dB (Use a Butterworth loss function in the analysis).

(b) Discuss the technique of frequency transformation applied to low-pass loss functions in the design of high-pass filters. Illustrate the answer using a second-order low-pass Butterworth loss function to produce a high-pass transfer function with a desired passband edge frequency of 1 kHz and $A_{\text{max}}$ equal to 3 dB.

Q17

(a) It is desired to produce a 1 kHz sinusoidal signal from a 1 kHz squarewave. Obtain the transfer function for a low-pass filter, which will produce a maximum attenuation of the fundamental signal of 1 dB. The third harmonic (3 kHz) should be attenuated by 12 dB (Butterworth loss functions tables are available for use in your analysis).

(b) Identify the circuit shown in figure 6. State one possible application for this circuit. Obtain the high-pass transfer function, assuming equal value resistances.
Figure 6

Q18

1.(a) Identify the order of the active filter circuit in Figure 7. Hence obtain a Butterworth approximation function, using the tables supplied, which will have a passband edge frequency \( \omega_p = 1000 \text{ rad/s} \). The frequency correction factor \( \varepsilon = 1 \).

(b) Show that the transfer function for this circuit is:

\[
\frac{E_3}{E_1} = \frac{1}{1 + sC_1R} \left\{ \frac{1}{R^2C_2C_3} \right\} \left[ \frac{1}{s^2 + \frac{2}{C_1R} + \frac{1}{R^2C_2C_3}} \right]
\]

Hint: (Apply nodal analysis separately to each circuit isolated by the buffer amplifier).

(c) Calculate suitable values for the three capacitors if the resistance \( R = 100 \text{ k}\Omega \).

Sketch the form of the frequency response for the two outputs at node 6.
Q19

a) Indicate how the order of a filter may be obtained using the expression for $A(\omega)$ and the following terms:
   - Maximum passband attenuation $A_{\text{max}} = 3 \text{ dB}$,
   - Minimum stopband attenuation $A_{\text{min}} = 28 \text{ dB}$,
   - Passband edge frequency $\omega_p = 1 \text{ k rs}^{-1}$, and
   - Stopband edge frequency $\omega_s = 10 \text{ k rs}^{-1}$.

   $$A(\omega) = 10 \log_{10} \left[ 1 + e^{\frac{-2}{\omega_p / \omega}} \right]$$

   [7 marks]

(b) Hence obtain a transfer function to meet the above specification.

(c) Discuss the method of frequency transformation used in designing bandpass filters. Use the first order Butterworth approximation function $A(\$) = \$ + 1$ to illustrate your answer. Include in your discussion any relevant formulae.

[10 marks]

Q20

(a) Obtain a transfer function for a third-order high-pass filter, which uses a Butterworth loss function, and which meets the following specification:
   - The maximum passband loss $A_{\text{max}} = 3 \text{ dB}$
   - The minimum stopband loss $A_{\text{min}} = 28 \text{ dB}$
   - The passband edge frequency $\omega_p = 6000 \text{ rs}^{-1}$
   - The stopband edge frequency $\omega_s = 2000 \text{ rs}^{-1}$

   [10 marks]

(b) A second-order IGMF bandpass active filter is shown in Figure 8. Show, by means of nodal analysis, how the transfer function is:

   $$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{s}{s^2 + s \frac{2}{CR_2} + \frac{1}{C^2 R_1 R_2}}$$

   [8 marks]

Calculate a value for the centre frequency $\omega_0$, the -3 dB bandwidth and the passband gain for the circuit values given.
A low-pass filter is required to meet the filter specification:
- The maximum passband loss $A_{\text{max}} = 0.5 \text{ dB}$
- The minimum stopband loss $A_{\text{min}} = 12 \text{ dB}$
- The passband edge frequency $\omega_p = 100 \text{ rad/s}$
- The stopband edge frequency $\omega_s = 400 \text{ rad/s}$

Determine the filter order $n$ using the relationship

$$n = \frac{\log_{10} \left[ \frac{10^{0.1A_{\text{max}}} - 1}{10^{0.1A_{\text{min}}} - 1} \right]}{2 \log_{10} \left( \frac{\omega_s}{\omega_p} \right)}.$$

Sketch the pole-zero diagram using the relationship between $n$ and the angle $\theta_k$, between the poles

$$\theta_k = \frac{360^\circ}{2n}.$$

Hence obtain a transfer function, which meets the specification.

Note: Butterworth tables are not supplied.

(b) Obtain circuit values for an equal component value Sallen and Key VCVS circuit configuration that would meet the specification in part (a).

Q23

(a) State one advantage and one disadvantage of using Chebychev polynomials in approximation loss function analysis.
(b) A bandpass filter is required for a particular application, which will tolerate ripple in the passband equal to 0.5 dB. The specification for the bandpass filter is:

- \( \omega_{bs1} \) = the lower stopband edge frequency = 688 rs\(^{-1}\)
- \( \omega_{bs2} \) = the upper stopband edge frequency = 1930 rs\(^{-1}\)
- \( \omega_{bp1} \) = the lower passband edge frequency = 970 rs\(^{-1}\)
- \( \omega_{bp2} \) = the upper passband edge frequency = 1370 rs\(^{-1}\)

The maximum passband attenuation \( A_{max} \) = 0.5 dB.

The minimum stopband attenuation \( A_{min} \) = 15 dB.

Obtain the bandpass transfer function, which will meet this specification. (Use the available Chebychev tables).

**Q24**

Draw the circuit diagram of a second-order bi-quad filter, which utilises three operational amplifiers. [4 marks]

Obtain an expression for the bandpass voltage transfer function for this configuration. [10 marks]

Such a transfer function is defined as

\[ G(s) = \]  

Select suitable resistive component values for this filter if equal value 100 nF capacitors are to be used in the design. Obtain an expression for the bandpass voltage transfer function for this configuration. [10 marks]

**Q25**

The loss-function for a low-pass Butterworth filter, expressed in dB is

\[ A(\omega) = 10 \log_{10} \left[ 1 + \left( \frac{\omega}{\omega_p} \right)^2 \right] \]

Using the above function, obtain expressions for the frequency correction factor and the order in terms of \( A_{max}, A_{min} \), the passband and stopband edge frequencies. [8 marks]

Determine a value for \( \varepsilon \) and \( \eta \) if the specification for an active low-pass filter is:

- Maximum passband attenuation \( A_{max} = 2 \) dB,
- Minimum stopband attenuation \( A_{min} = 12 \) dB,
- Passband edge frequency \( \omega_p = 1 \) kr\(^{-1}\),
- Stopband edge frequency \( \omega_s = 2 \) kr\(^{-1}\). [12 marks]

**Q26**

The bandpass circuit in Fig. 10 has a transfer function. Determine suitable component values for \( C, R \) and \( R \) which will. The required bandpass frequency transformation is: where B is the -3 dB bandwidth that can be expressed as

The normalised low pass stopband frequency is defined as
Sketch a Sallen and Key equal-value component circuit-diagram that could be used to implement the transfer function so derived (circuit values not required).

Then the corresponding 3rd order normalised loss function is

\[ A(s) = (s + 1)(s^2 + s + 1) \]

\[ H(s) = \frac{1}{A(s)} = \frac{1}{(s + 1)(s^2 + s + 1)} \]

To find the attenuation use the following

\[ A(s) = 10 \log [1 + 0.585.3] \]

\[ A(s) = 10 \log [1 + 0.585x729] = 26.3 \text{ dB} \]

The angle is calculated as

**Q27**

Show that the transfer function \( H(s) \) for the circuit shown in Fig 11 is:

\[ H(s) = \frac{1}{(s^2 + 3s + 3)} \]

Sketch the pole-zero plot. Hence evaluate the magnitude of the transfer function at a frequency of \( 4 \text{ rad s}^{-1} \).

A unit impulse signal is applied to the second order active filter shown in Fig 11. Obtain an expression for the output voltage and hence evaluate the output voltage after an elapsed time of one second.

\( R1 = 0.5 \Omega, R2 = 1 \Omega, C1 = C2 = 1 \text{ F} \).

To find the attenuation at \( \omega \), use the following

\[ H(s) = \frac{1}{A(s)} = \frac{1}{10 \log[1 + \left( \frac{\omega}{\omega_p} \right)^2]} \approx \frac{1}{10 \log[1 + \left( \frac{6000}{2000} \right)^6]} = -28 \text{ dB} \]
Q28

A state-variable filter is shown in Fig. 12. Obtain the bandpass transfer function \( \frac{V_o(s)}{V_i(s)} \). Assuming a value for \( R = 10 \, k\Omega \), calculate a value for \( C \) which will set the centre frequency to a value \( f_0 = 10 \, \text{kr}^{-1} \). Calculate suitable values for \( R_a \) and \( R_b \), which will produce a Q-factor of 50.

[14 marks]

A 4th order bandpass is required whose Butterworth transfer function is:

Calculate suitable component values for two cascaded state-variable bandpass circuits whose overall transfer function is

[14 marks]

Solution

For the gain determining elements:

\[ Q = 50 = \frac{1}{3}(1 + \frac{R_4}{R_3}) \] or \[ 150 = 1 + \frac{R_4}{R_3} \] or \[ R_4/R_3 = 149 \]

i.e. \( R_4 = 149 \, R_3 \). Assume a value for \( R_3 = 1 \, k\Omega \) so that \( R_4 = 149 \, k\Omega \). Using these values the transfer function is \( (s) = BW 31.83 \, \text{Hz} \).

Since the transfer function in factored form indicates a BP with centre frequency = 1, then cascading two such circuits as in the first section will suffice. The damping of each section is determined by the resistors \( R_3, R_4, R_3', \) and \( R_4' \).

The circuit values can be calculated as follows:

Since \( \omega_0 = 1 \) and selecting a value for \( C = 10 \, \mu\text{F} \) and \( R = 100 \, k\Omega \) and comparing coefficients of the \( s \) terms gives then from \( R_3 = R_4 \) i.e. \( 92R_3 = R_4 \). Choose \( R_3 = 1 \, k\Omega \) & \( R_4 = 2.92 \, k\Omega \) similarly for the second stage or suitable values \( R_4' = 623 \, \Omega \) and \( R_3' = 1 \, k\Omega \).

So for each other. The transfer function for the Bandpass output is

For \( \omega_0 = 1/C = 10 \, \text{kr}^{-1} \) \( 6 \, C = 10 \, \mu\text{F} \).

A state-variable filter, which uses three operational amplifiers is shown in Fig.13. Obtain the low-pass transfer function \( H(s) \).

[10 marks]

for the gain determining elements as

\[ Q = 50 = \frac{1}{3}(1+R_4/R_3) \] i.e.

\[ 150 = 1 + \frac{R_4}{R_3} \] or \[ R_4/R_3 = 149 \] i.e. \( R_4 = 149 \, R_3 \)

Assume a value for \( R_3 = 1 \, k\Omega \) so that \( R_4 = 149 \, k\Omega \)

Using these values the transfer function is

\[ H(s) = \]

\[ BW = 31.83 \, \text{Hz} \]

Since the transfer function in factored form indicates a BP with centre frequency = 1, then cascading two such circuits as in the first section will suffice. The damping of each section will be determined by \( R_3 \) and \( R_4 \) and \( R_3' \) and \( R_4' \).

The circuit values can be calculated as follows:
Since $\omega_o = 1$ and selecting a value for $C = 10 \, \mu F$ and $R = 100 \, \Omega$ and comparing coefficients of the $s$ terms gives then from (1)

$$R3 = R4$$

i.e. $2.92R3 = R4$

Choose $R3 = 1 \, k\Omega$  $R4 = 2.92 \, k\Omega$

similarly for the second stage or suitable values $R4' = 623 \, \Omega$ and $R3' = 1 \, k\Omega$  $R4' = 623 \, \Omega$ of each other. The transfer function for the Bandpass output is For $\omega_o = 1/CR = 10 \, k\Omega$  $= 10 \, nF$.

**Q29**

A low -pass Butterworth filter is required to meet the following specifications:

- Maximum passband attenuation $A_{max} = 2 \, dB$,
- Minimum stopband attenuation $A_{min} = 12 \, dB$,
- Passband edge frequency $f_p = 1 \, kHz$, and
- Stopband edge frequency $f_s = 2 \, kHz$.

Determine

(a) The order $n$ of the filter,
(b) $\varepsilon$, the frequency scaling factor, and
(c) The transfer function.

Sketch the pole zero pattern for this transfer function.  

Determine the transfer function for a high-pass filter which would have $A_{max} = 2 \, dB$, $A_{min} = 12 \, dB$ and a passband edge frequency $= 1 \, kHz$. (Table 1 gives the Butterworth normalised loss functions)

**Q30**

Discuss the technique of frequency transformation in obtaining a transfer function for a high-pass filter to meet desired hp specifications and using available low-pass loss function tables.

The specifications desired for a high-pass filter are:

- Maximum passband edge attenuation $A_{max} = 2 \, dB$,
- Minimum stopband edge attenuation $A_{min} = 20 \, dB$,
- Passband edge frequency $\omega_p = 3 \, krs^{-1}$
- Stopband edge frequency $\omega_s = 1.5 \, krs^{-1}$

By using an appropriate frequency transform, obtain a Butterworth high-pass transfer function which will meet these specifications.

**Q31**

A low-pass filter transfer function is given as:

$$H(s) = \frac{1}{s + 1}$$

Using frequency transformation techniques on this transfer function, demonstrate how to obtain:
i) A Chebychev high-pass filter with $\omega_p = 10 \text{ rs}^{-1}$, and
ii) A Chebychev bandpass filter with $\omega_o = 1 \text{ krs}^{-1}$ and -3 dB bandwidth = 100 rs$^{-1}$.

**Q32**

Sketch the circuit diagram of a second-order filter circuit that could be used in the equalisation section of an audio mixing desk. Such a circuit would have low-pass, high-pass and bandpass outputs. Indicate which components determine the frequency characteristics of the filter. State briefly how you would change the configuration from a Butterworth to a Chebychev filter.

[10 marks]

**Q33**

a) Indicate how an expression for the order of a filter may be obtained using the following terms:
- Maximum passband attenuation $A_{\text{max}}$,
- Minimum stopband attenuation $A_{\text{min}}$,
- Passband edge frequency $\omega_p$, and
- Stopband edge frequency $\omega_s$.

You may use a Butterworth or a Chebychev function in your analysis.

[6 marks]

b) Obtain an appropriate loss -function $A(\omega)$ which will meet these specifications. Hence calculate the actual loss in dB produced by the filter at the stopband edge frequency.

Show how this transfer function could be implemented using an active filter. Circuit values are not required but indicate which components determine the passband frequency. Discuss briefly which components determine whether the circuit configuration is a Chebychev or a Butterworth type of filter.

[8 marks]

**Q34**

Discuss a graphical technique for locating the poles of a Chebychev approximation loss-function on an s-plane plot.

[4 marks]

The bandpass circuit configuration in Fig. 14 has a transfer function. Determine suitable component values for $C$, $R$ and $R$ which will The required bandpass frequency transformation is :where $B$ is the -3dB bandwidth which can be expressed as

The normalised low pass stopband frequency is defined as

**Q35**

An anti-aliasing low-pass filter is required to meet the following filter specifications:

- The maximum passband loss $A_{\text{max}} = 2 \text{ dB}$
- The minimum stopband loss $A_{\text{min}} = 23 \text{ dB}$
- The passband edge frequency $\omega_p = 3.10 \text{ rs}^{-1}$
- The stopband edge frequency $\omega_s = 9.10 \text{ rs}^{-1}$
Obtain an appropriate loss function $A(\$)$ which will meet these specifications. Hence calculate the actual loss in dB produced by the filter in dB the stopband edge frequency. [12 marks]
From the transfer function $H(\$)= 1/A(\$)$, calculate the phase shift produced at the passband edge frequency.

Sketch a Sallen and Key equal-value component circuit diagram which could be used to implement the transfer function so derived. (circuit values not required)

Solution

Then the corresponding 3rd order normalised loss function is

$$A(\$) = (s + 1)(s^2 + s + 1)$$

$$H(\$) =$$

To find the attenuation use the following

$$A(\$) = 10 \log[1 + 0.585x3]$$

$$A(\$) = 10 \log[1 + 0.585x729] = 26.3 \text{ dB}$$

The angle is calculated as

**Q36**

Discuss a graphical technique for locating the poles of a Chebychev approximation loss-function on an s-plane plot.

The bandpass circuit configuration in Fig. 15 has a transfer function. Determine suitable component values for $C$, $R$ and $R$ which will The required bandpass frequency transformation is: where $B$ is the –3 dB bandwidth which can be expressed as bp bp1. The normalised low pass stopband frequency is defined as

POLE-ZERO PLOT OF H(s)

THE STEP RESPONSE

The output voltage $V$ is $V(s) = H(s)V(s)$ so for a step i/p

Let $s + 1/C R$

1 - $t/C (R + R)$

Obtain the voltage transfer function $H(s)$ and plot the pole-zero constellation for $H(s)$ . Hence using this diagram, show that the network in Fig.2 is an ALL-PASS filter [12 marks]

Explain the terms minimum- phase function and non-minimum -phase function . Show how the network in Fig.16 is a non-minimum phase network and state one application for this network.
The Laplace Transform of a function \( f(t) \) is defined as:

\[
L[f(t)] = \int_0^\infty f(t)e^{-st} \, dt
\]

Show that a capacitor charged to \( V \) volts, can be represented as a capacitive reactance \( 1/sC \) in series with a voltage source \( V/s \).

[6 marks]

A 10 H coil with a measured resistance of 1600 \( \Omega \) is connected across the terminals of a capacitor, which had been previously charged to 100 V. Obtain an expression (as a function of time) for the voltage across the coil after the capacitor has been connected. Calculate a value for this voltage at a time \( t = 1 \) second has elapsed.

[14 marks]

**Q37**

(a) Show that the unit step response \( G(t) \), of a simple CR low-pass filter, is the time integral of its impulse response.

[6 marks]

(b) For the circuit shown in Fig.17, steady state conditions are established when \( S \) is opened. \( S \) is then closed. Derive an expression for the voltage \( V(s) \) across \( C \). Apply the initial and final value theorems to obtain a value for \( V(t) \) at time \( t = 0 \) and seconds.

[14 marks]

\[ E = 1 \text{ V}; \quad R = R = 1; \quad L = 1 \text{ H}; \quad C = 1 \text{ F}. \]

Draw a block diagram of a PLL.

Obtain a value for the open loop gain \( k \) for the circuit parameters given. Hence show, from first principles, how the closed loop gain is For the circuit in Fig 1 and using mesh analysis show that current in \( R = 0 \) (the balance condition ) obtains for the condition

[8 marks]

POLE-ZERO PLOT OF \( H(s) \)

**THE STEP RESPONSE**

The output voltage \( V \) is \( V(s) = H(s)V(s) \) so for a step i/p

Let \( s + 1/C R \)

\[ 1 - t/C \left( R + R \right) \]

b) A 1 volt step is applied to this circuit. Sketch to scale the output voltage over the period 0 to 5 seconds for the following circuit values :

\( R = 10 \text{ k}\Omega, \quad R = 100 \text{ k}\Omega, \quad C = 10 \text{ nF} \) [12 marks]

a) Derive the voltage transfer function \( H(s) \) and sketch the pole-zero plot for the network in Fig.2

[6 marks]
b) Sketch to scale the output voltage over the period 0 < 3t < 15 s when the following signals are applied to the input:

i) a unit step, and

ii) a unit impulse [12 marks]

c) Show that the differential of the step response from i) is the impulse response obtained in ii). [2 marks]

R = 10 kΩ, C = 10 nF

To find values for A and B use the partial fraction method

The transfer function \( H(s) = \frac{1}{C R} \frac{s}{s + 1/CR} \)

Let us consider the step response for the \( CR \) low-pass filter. The Laplace transform of a step is \( 1/s \) so that we can write the output response \( V(s) = H(s) \cdot \frac{1}{s} \).

POLE-ZERO PLOT OF \( H(s) \)

The impulse response is the transfer function since the transform of an impulse function is one so we can write \( V221(s) = H(s) \).

We can write that \(-3t/CR \cdot V(3t1) = 1/CR \cdot e^{\omega u} \).

Which is the result we obtained for the impulse response i.e. differentiate a step response 1 and you get the impulse response /

Obtain the voltage transfer function \( H(s) \) and plot the pole-zero constellation for \( H(s) \).

Hence using this diagram, show that the network in Fig.2 is an ALL-PASS filter [12 marks]

Explain the terms minimum-phase function and non-minimum-phase function.

Show how the network in Fig.2 is a non-minimum phase network and state one application for this network.

MINIMUM AND NON-MINIMUM PHASE FUNCTIONS

A system with left-hand zeros only are called minimum-phase functions. It can be seen from a pole-zero plot of such functions that the angle of the transfer function will vary over a much angle than a system which has right-hand zeros. Considering the all-pass network in Fig.2 the angle for the transfer function \( H(s) \) is given as One application for this network is where a signal is suffering from an unwanted phase lag then the network can supply enough additional phase lag to bring the total angle to 0.

From the table of Butterworth loss functions we obtain a 4th order polynomial.

\[ A(s) = (s + 0.765s + 1)(s + 1.848s + 1) \]

De-normalising and LP to HP transforming substituting for \( 1 \) as \( b = \omega _{hp}/1/s = (0.764) = 2800/s \)

substituting this into the required high-pass approximating function which, when inverted, gives the required H-P transfer function. 0( )
\[ A(s) = 0 \{ 1[(2800/s) + 0.765(2800/s) + 1][(2800/s) + 1.848(2800/s) + 1] \} \]

Inverting this to obtain the required transfer function.

\[ H(s) = (s + 2142s + 2800)(s + 5174s + 2800) \]

The passband frequency is determined by \( C, R_c \) will determine whether we have a Butterworth or a Chebychev response. The damping is set by \( k = 1 + R_a/R_b \) and \( 1 + R_c/R_d \).

Draw the circuit configuration for an equal component high-pass Sallen and Key active filter. Obtain the transfer function \( H(s) \) for this circuit and show that passband edge frequency \( \omega_p \) and damping \( \omega_p/Q \) are \( 1/CR \) and \( (3 - K)/CR \) resp. You may assume the operational amplifier is ideal.

HIGH-PASS SPECIFICATIONS = 0 1.5kΩ 3 kΩ EQUIVALENT LOW-PASS substituting values From the table of Butterworth loss functions we obtain a 4th order polynomial. De-normalising and LP to HP transforming substituting for as substituting this into the required high-pass approximating function which, when inverted, gives the required H-P transfer function.

\[ A(s) = \{(2800/s) + 0.765(2800/s) + 1\}\{(2800/s) + 1.848(2800/s) + 1\} \]

Inverting this to obtain the required transfer function.

\[ H(s) = (s + 2142s + 2800)(s + 5174s + 2800) \]

The passband frequency is determined by \( C, R_c \) will determine whether we have a Butterworth or a Chebychev response. The damping is set by \( k = 1 + R_a/R_b \) and \( 1 + R_c/R_d \).

Draw equivalent circuits for a capacitor \( C \) with an initial voltage volts and an inductor \( L \) with an initial current \( I = 20 \) amps

Steady state conditions are established for the circuit in Fig.1 with opened. S is then closed. Derive an expression for the voltage \( V_{2o} \) across \( R \) Obtain a value for \( V \) at time = 0 and 98 seconds by applying the Initial and Final value theorems

Applying Thévenin's theorem to the circuit yields The initial conditions are calculated from Fig 1b above by considering the capacitor \( C \) as an o/c and \( L \) as a s/c. Thus The initial voltage on the capacitor is The initial condition for the inductor is calculated: The equivalent s-plane circuit for \( t > 0 \) is Thus applying KVL to the circuit in Fig.2 Applying the initial value to this equation gives a value for \( V/2R_2 \) and the Final value theorem Value at \( t = 98 \) \( \lim sF(s) = \lim f(t) \) Value at \( 98 \) \( \lim sF(s) \) a)

Derive the voltage transfer function \( H(s) \) and sketch the pole-zero plot for the network in Fig.2

b) Sketch to scale the output voltage over the period \( 0 < 3t < 15 \) s when the following signals are applied to the input : i) a unit step and ii) a unit impulse [12 marks]

c) Show that the differential of the step response from i) is the impulse response
obtained in ii).

[2 marks]

$= 10 \, \text{k}\Omega, \, C = 10 \, \text{nF}$ Fig. 2 To find values for $A$ and $B$ use the partial fraction method

Fig 4 Step response of low-pass CR net Fig. 5 \3low-pass CR filter The transfer function $H(s) = \text{The required transfer function is: Let us consider the STEP response for the CR low-pass filter. The Laplace transform of a step is } 1/s \text{ so that we can write the output response } V_1(s) :

IMPULSE RESPONSE The impulse response is the transfer function since the transform of an impulse function is one so we can write We can write that Fig 7 Impulse response of Low-pass CR net If we differentiate the step response we get the result: Which is the result we obtained for the impulse response i.e. differentiate a step response and you get the impulse response

b) A 1 volt step is applied to this circuit. Sketch to scale the output voltage over the period 0 to 5 seconds for the following circuit values:

$R = 10 \, \text{k}\Omega, \, R = 100 \, \text{k}\Omega, \, C = 10 \, \text{nF}$

[12 marks]

POLE-ZERO PLOT OF $H(s)$

THE STEP RESPONSE

The output voltage $V$ is $V(s) = H(s)V(s)$ so for a step i/p

Let $s + 1/C \, R$

$1 - t/C \, (R + R)$

Obtain the voltage transfer function $H(s)$ and plot the pole-zero constellation for $H(s)$. Hence using this diagram, show that the network in Fig. 2 is an ALL-PASS filter [12 marks]. Explain the terms minimum-phase function and non-minimum-phase function. Show how the network in Fig. 2 is a non-minimum phase network and state one application for this network.