Chapter 7

The Biquad filter

The biquad configuration is a useful circuit for producing bandpass and low-pass responses, which require high Q-factor values not achievable with the VSVS and the IGMF circuits. The Biquad and the state variable filter circuit configuration can have Q-factor values of 400 or greater. This circuit, whilst not as useful as the state variable, nevertheless has certain applications. It is easily tuneable using single resistor tuning (normally a stereo or ganged potentiometer). It can be configured to produce a Butterworth or a Chebychev response by changing the damping \((1/Q)\).

Figure 7.47 shows a typical three-operational amplifier circuit. The transfer function is obtained by considering the bandpass output first. The low-pass is easily obtained thereafter. The Biquad active filter consists of a leaky integrator, an integrator and a summing amplifier. Let \(Z_f\) be the parallel combination of \(R_2\) and \(C_1\):

\[
Z_f = \frac{R_2}{1 + sC_1R_2}
\]

The transfer function for the last stage is defined as:

\[
\frac{V_4}{V_3} = -\frac{R_4}{R_4}
\]

7.1

In addition, for the second last stage:

\[
\frac{V_3}{V_2} = -\frac{1}{sC_2R_3}
\]

7.2
The first op-amp is a leaky integrator and we can write for the output voltage as:

\[ V_2 = -\frac{Z_f}{R_6} V_4 - \frac{Z_f}{R_1} V_1 \]

If we substitute for \( V_4 \) and \( V_2 \) from equations 7.1 and 7.2, we can write

\[ V_4 = -\frac{R_5}{R_4} V_3 \]

But

\[ V_3 = -\frac{1}{sC_2 R_3} V_2 \]

\[ V_4 = \frac{R_5}{R_4} \frac{1}{sC_2 R_3} V_2 \]

Substituting 7.6 into 7.3 yields

\[ V_2 = -\frac{Z_f}{R_6} R_5 \frac{1}{sC_2 R_3 R_4} V_2 - \frac{Z_f}{R_1} V_1 \]

\[ V_2 [1 + \frac{Z_f}{R_6} \frac{R_5}{sC_2 R_3 R_4}] = -\frac{Z_f}{R_1} V_1 \]

\[ \frac{V_2}{V_1} = -\frac{\frac{Z_f}{R_1}}{1 + \frac{Z_f R_5}{sC_2 R_3 R_4 R_6}} \]

\[ \frac{V_2}{V_1} = -\frac{\frac{R_2}{1 + sC_1 R_2}}{\frac{R_2}{1 + sC_1 R_2} - \frac{R_3}{1 + sC_2 R_3 R_4 R_6}} \]

\[ \frac{V_2}{V_1} = -\frac{\frac{R_2}{1 + sC_1 R_2} + \frac{R_3 R_5}{sC_2 R_3 R_4 R_6}}{1 + sC_1 R_2} \]
\[ V_2 = \frac{R_2}{R_1} s C_2 R_3 R_4 R_6 \]
\[ V_1 = s^2 C_1 C_2 R_2 R_3 R_4 R_6 + s C_2 R_3 R_4 R_6 + + R_2 R_3 \]
\[ V_2 = - \frac{R_2}{s R_1} - \frac{C_2 R_3 R_4 R_6}{s C_1 C_2 R_2 R_3 R_4 R_6} \]
\[ V_1 = \frac{1}{s^2 + s \frac{1}{C_1 R_2} + \frac{R_2}{C_1 C_2 R_3 R_4 R_6}} \]

If \( R_4 = R_5 \):
\[ V_2 = - \frac{s}{C_1 R_1} \]
\[ V_1 = \frac{1}{s^2 + s \frac{1}{C_1 R_2} + \frac{1}{C_1 C_2 R_3 R_6}} \]

The standard form for a bandpass second order function is
\[ V_2 = - s K \frac{\omega_p}{Q} \]
\[ V_1 = \frac{1}{s^2 + s \frac{\omega_p}{Q} + \omega_p^2} \]

We may write expressions for the passband edge frequency, \( Q \)-factor and gain in terms of the circuit components by comparing coefficients. The gain at the resonant frequency is
\[ H(0) = K = \frac{R_2}{R_1} \]

The resonant frequency is
\[ \omega_{o}^2 = \frac{1}{C_1 C_2 R_3 R_6} \]
The $Q$-factor and the resonant frequency are not independent in this circuit. For high frequencies, the bandwidth will be the same as that for low frequencies. This, in general, is not a desirable feature. For example, in an audio mixing desk, the equalising section would use a state-variable circuit where the bandwidth changes with the higher frequencies. The tuning procedure for the biquad BP is as follows:

1) Select values for $C_1$, $C_2$ and $R_5$, 
2) Adjust the resonant frequency $\omega_p$ by varying $R_3$ set the gain by $R_1$, 
3) The $Q$-factor is set by $R_2$. as opposed to iterative where one has to keep adjusting the component values to get the desired value, and 
4) The $Q$-factor is set by $R_2$.

**Equal value component**

If we set $C_1 = C_2 = C$ and $R_3 = R_6 = R$, then the equations are greatly simplified

$$V_2 = -\frac{s \frac{1}{CR_1}}{s^2 + s \frac{1}{CR_2} + \frac{1}{C^2 R^2}}$$

$$H(0) = K = \frac{R_2}{R_1}$$

$$\omega^2_0 = \frac{1}{C^2 R^2}$$

$$Q = \sqrt{\frac{R_2^2}{R^2}} = \frac{R_2}{R}$$

$$BW = \frac{1}{2\pi CR_2}$$
Figure 7.48: Biquad filter response.