Digital Communications Engineering 1

(COMM2108)

Baseband Signal Transmission

Baseband Signal Transmission

- All transmission media attenuate signals to a greater or lesser extent.
- For long communication paths the *attenuation loss* may be so great that the sensitivity of the receiver may be inadequate to detect the signal.
- A piece of communications equipment known as a *repeater* is used to boost the signal level and to compensate for attenuation losses.
- The path between adjacent repeaters is known as a *hop*.
- Long distance communication usually involves many such hops.

Baseband Signal Transmission

- There are essentially two types of repeater:
  - *Amplifying repeaters*
  - *Regenerative repeaters*.
- For analogue communication amplifying repeaters are used.
- For digital communication both types of repeater can be used. However, the regenerative type is by far the most common.
- The error performance for the two repeater types are compared for digital signals only.

Baseband Signal Transmission

- Considering the use of linear amplifiers as repeaters.
- Assuming a m-hop link with polar signaling where the pulse amplitudes are $\pm A$ volts.
- At the input to the first amplifier, the signal voltage is

$$\pm \alpha A + n_1(t)$$

- where $\alpha<1$ is the attenuation factor and $n_1(t)$ is the additive white Gaussian noise (AWGN) with an average noise power or variance σ^2.

Baseband Signal Transmission

- Assuming that the amplifier has a voltage gain

$$G_v = 1/\alpha$$

- In other words, the gain of the amplifier is just sufficient to balance out the attenuation loss.

- At the output of the first amplifier, there is a signal voltage

$$\pm A + G_v n_1(t)$$

- Assuming each hop incurs the same attenuation,the input to the second amplifier will be

$$\pm \alpha A + n_1(t) + n_2(t)$$

Baseband Signal Transmission

- After m-hops, the signal voltage will be

$$\pm\alpha A + n_1(t) + n_2(t) + \ldots\ldots + n_m(t)$$

- Assuming that the noise sources are statistically independent, the total average noise power will be

$$\sigma^2 = \sum_{i=1}^{m} \sigma_i^2$$

$$= m\sigma^2$$

- where it is assumed that $\sigma_i = \sigma$ for all i.

Baseband Signal Transmission

- Using the earlier result for binary polar signaling

$$P_e = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

- where E_s is the average symbol energy, i.e. $E_s = A^2T$. The average noise power spectral density N_0 may be calculated by assuming the use of the minimum Nyquist signaling bandwidth.

$$\begin{aligned} N_0 &= \frac{m\sigma^2}{BW} \\ &= \frac{m\sigma^2}{1/T} \\ &= m\sigma^2 T \end{aligned}$$

Baseband Signal Transmission

- Substituting for E_s and N_0 gives an expression for the average probability of error P_e after m-hops.

$$P_e = Q\left(\sqrt{\frac{2A^2}{m\sigma^2}} \right)$$

- It is worth noting that the noise power accumulates along a communications link containing amplifying repeaters.
- The larger the value of m the larger the value of P_e and hence the more errors that will occur.

Baseband Signal Transmission

- Next the use of regenerative repeaters is considered.
- When using a regenerative repeater the noise does not accumulate from repeater to repeater as a new noiseless pulse is regenerated at the output of each repeater.
- In other words, the pulse is being continually “cleaned-up” as it propagates along the communications link.
- However, any errors that occur on the link are propagated down the link. As a result the probability of errors accumulates linearly over the communications link.

Baseband Signal Transmission

- Using the analysis developed for the case of amplifying repeaters, the average probability of error P_e after m-hops is

$$P_e = mQ\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

$$= mQ\left(\sqrt{\frac{2A^2}{\sigma^2}}\right)$$

- From the hand-out, it can be seen that the accumulation of errors grows more rapidly on analogue links than on digital links.

Baseband Signal Transmission

- Having considered the effects of attenuation on the transmission of digital baseband signals, it is now necessary to consider the effects of *dispersion*.
- Dispersion is a consequence of the fact that the frequency components of a signal propagate at slightly different velocities in a communications medium.
- At the receiver, the various frequency components of a signal will be differentially delayed with respect to each other (i.e. the frequency components will arrive at the receiver at slightly different times).
- In the case of a pulse propagating through a medium, dispersion will cause the pulse to become distorted or “smeared out” in time.

Baseband Signal Transmission

- In general, whenever pulses are transmitted through a bandlimited communications channel, the pulse will become distorted or “smeared out” in time. This is known as *dispersion*.
- As a consequence of being smeared out, the energy in the pulse is no longer confined to its own specific time slot and spreads out to occupy adjacent time slots.
- As a result, the pulses will tend to overlap each other and eventually become indistinguishable at the receiver.
- This situation is known as *InterSymbol Interference,* or *ISI* for short, and leads to errors on digital communications links.

Baseband Signal Transmission

- Considering the elements of a baseband digital transmission system.
- The input signal consists of a binary data sequence $\{b_k\}$ with a bit duration of T_b seconds.
- This input sequence is applied to a pulse generator which generates the desired line code format.
- The output is a digital signal with the following form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k f(t - kT_b)$$

- where $f(t)$ denotes the basic pulse shape and a_k is the amplitude coefficient that depends on the input data and the format used.

Baseband Signal Transmission

- The digital signal $x(t)$ passes through a transmitting filter $H_T(f)$, a communications channel represented by $H_C(f)$, and finally a receiving filter $H_R(f)$.
- The output from the receiving filter is

$$y(t) = \sum_{k=-\infty}^{\infty} a_k \, p(t - kT_b)$$

- where $p(t)$ is the response of the cascaded connection of transmitting filter, communications channel, and receiving filter, to the input pulse $f(t)$.

Baseband Signal Transmission

- In the frequency domain

$$P(f) = H_R(f)H_C(f)H_T(f)F(f)$$
$$= H(f)F(f)$$

- where $H(f)$ is the overall equivalent system transfer function

$$H(f) = H_R(f)H_C(f)H_T(f)$$

- The receiving filter output $y(t)$ is sampled at a time t_i = iT_b where i is an integer.

$$y(t_i) = \sum_{k=-\infty}^{\infty} a_k p(iT_b - kT_b)$$

$$= a_i p(0) + \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(iT_b - kT_b)$$

Baseband Signal Transmission

- The first term $a_i p(0)$ represents the desired response, i.e. the output produced by the i^{th} transmitted bit.
- The second term represents the residual effect of all the other transmitted bits on the decoding of the i^{th} bit.
- This residual effect is *InterSymbol Interference (ISI).*
- Nyquist proposed three methods by which ISI may be eliminated.
- However, only one of these methods will be considered here, namely *Nyquist's First Criterion* for the elimination of ISI.

Baseband Signal Transmission

- *Nyquist's First Criterion* requires that the received pulse is controlled such that

$$p(iT_b - kT_b) = \begin{cases} 1 & for\ i = k \\ 0 & for\ i \neq k \end{cases}$$

- If this condition is satisfied, the receiver output simplifies to

$$y(t_i) = a_i$$

- which implies zero ISI and will ensure perfect (i.e. distortionless) reception in the absence of noise.

Baseband Signal Transmission

- A good choice for $p(t)$ that satisfies Nyquist's First Criterion is the *sinc* function

$$p(t) = \mathrm{sinc}\left(t/T_b \right)$$

- This function $p(t)$ has its peak value at $t = 0$ and passes through zero at integer multiples of the bit duration T_b.
- Consequently, if the received waveform $y(t)$ is sampled at times $t = 0, \pm T_b, \pm 2T_b, \ldots$, the pulses defined by $p(t-iT_b)$ will not interfere with each other.

Baseband Signal Transmission

- In addition to eliminating ISI, the *sinc* form for $p(t)$ minimises the bandwidth requirement for the transmission of $p(t)$.
- Since

$$P(f) = \Im^{-1}\{p(t)\}$$
$$= T_b \prod (fT_b)$$

- $P(f)$ represents an ideal low pass filter (LPF) with a response of T_b and a bandwidth $B = 1/2T_b$.
- In terms of the bandwidth B,

$$P(f) = \frac{1}{2B} \prod \left(f/2B \right)$$

Baseband Signal Transmission

- With *sinc* pulses, the bandwidth $B = 1/2T_b$ will support a bit rate of $R_b = 1/T_b$.
- As a result the spectral efficiency $\eta = 2$ *bits/sec/Hz*, this is known as the *Nyquist minimum bandwidth constraint* as it represents the minimum bandwidth required for the transmission of digital pulses.
- Although the choice of a *sinc* function for the pulse shape $p(t)$ achieves economy in bandwidth in solving the problem of ISI, there are two major practical difficulties.

Baseband Signal Transmission

- The first practical difficulty is that the pulse shape is physically unrealisable. It is not possible to construct a filter with an infinitely sharp cut-off.
- Secondly, the $sinc$ function is very intolerant of timing errors at the sampling instants. This is due to the very steep slope of the $sinc$ function as it passes through its zero crossing point. As a result, a small error on the timing of the sampling instant will lead to a large ISI error.
- Also the $sinc$ function decays slowly with the amplitude peaks decaying at a rate $1/|t|$.
- If the $sinc$ function could be made to decay more rapidly, it would reduce the effects of timing errors.

Baseband Signal Transmission

- Because of these short-comings other pulse shapes have been proposed, usually at the expense of an increased bandwidth.
- For example, the *raised cosine pulse* will be considered here.
- The spectrum of the raised cosine pulse consists of a flat portion and a sinusoidal roll-off portion.
- The raised cosine pulse has the following form

$$P(f) = \begin{cases} \dfrac{1}{2B} & |f| < f_1 \\ \dfrac{1}{4B}\left\{1 + \cos\left[\dfrac{\pi\left(|f| - f_1\right)}{2B - 2f_1}\right]\right\} & f_1 \leq |f| < 2B - f_1 \\ 0 & |f| \geq 2B - f_1 \end{cases}$$

Baseband Signal Transmission

- The frequency f_1 and the bandwidth B are related by

$$\alpha = 1 - \frac{f_1}{B}$$

- where α is called the roll-off factor.
- The time response $p(t)$ is given by

$$\begin{aligned} p(t) &= \Im^{-1}\{P(f)\} \\ &= \operatorname{sinc}(2Bt)\frac{\cos(2\pi\alpha Bt)}{1-16\alpha^2B^2t^2} \end{aligned}$$

Baseband Signal Transmission

- This function consists of two factors:
- (i) The $sinc$ function ensures that there are zero crossing points at integer multiples of T_b to eliminate ISI.
- (ii) The second term ensures a fast roll-off for the function as it decreases as $1/|t|^2$.
- The tails of the pulse are well below those for the ideal case which ensures the system will be less sensitive to sampling time errors.
- The bandwidth B_{RC} of the raised cosine pulse is

$$
\begin{aligned}
B_{RC} &= 2B - f_1 \\
&= 2B - B(1-\alpha) \\
&= (1-\alpha)B
\end{aligned}
$$

Baseband Signal Transmission

- In digital systems, a process known as *equalisation* is used to compensate for channel induced distortion.
- An special filter known as an *equaliser filter* is used.
- A common choice for the equaliser filter is the *transversal filter* which comprises a tapped delay line.
- The tap outputs are multiplied by gain factors (i.e the tap weights) and are summed together to produce the filter output.
- The main task in equalisation is to find the values for the tap weights that will eliminate the ISI at the output.

Baseband Signal Transmission

- As the channel characteristics are subject to change, the equaliser filter is usually adaptive in that the tap coefficients are continually adjusted to meet the changing channel conditions.
- There are two types of automatic equalisation:
  - *Preset Equalization*
  - *Adaptive Equalization*

Baseband Signal Transmission

- In *preset equalisation*, a known sequence of bits, known as a *training sequence*, is transmitted at the beginning of a communications session.
- The equaliser uses this sequence to construct a model of the channel which can then be used to counteract the effects of channel by setting the tap coefficients to produce the inverse channel transfer function.
- In *adaptive equalisation*, the tap coefficients are regularly updated to follow the changes in the channel characteristics.
- Training sequences are transmitted on a regular basis to facilitate the equalisation process.

Baseband Signal Transmission

- The effects of noise and ISI on the performance of a baseband digital transmission system can be observed by displaying the received waveform on an oscilloscope.
- By displaying multiple sweeps, where each sweep is triggered by a clock signal, a display known as an *eye-pattern* emerges.
- The interior of the eye is known as the *eye opening.*
- Under ideal conditions the eye will be open. However, if noise or ISI is present the eye will close.
- The eye pattern can give very useful qualitative information on the performance of the communications system.

Baseband Signal Transmission

- Specifically, the eye pattern can provide the following information.
- The width of the eye opening determines the time interval over which the received signal can be sampled without error from ISI. The optimum sampling instant is when the eye opening is at its widest.
- The sensitivity to timing errors is given by the slope of the open eye at the zero crossing point.
- The noise margin of the system is given by the height of the eye opening.