Dublin Institute of Technology

School of Electronic and Communications Engineering

Optical Communications Systems

Dispersion in Optical Fibre (II):

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Chromatic Dispersion

Chromatic Dispersion

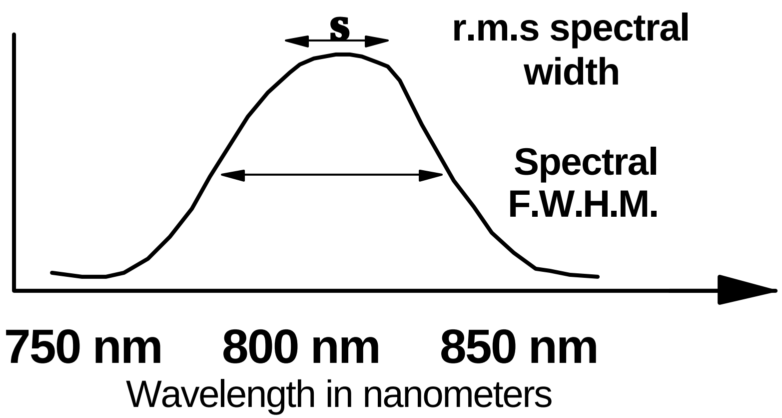
Chromatic dispersion is actually the sum of two forms of dispersion

Material Dispersion	+	Waveguide Dispersion	=	Chromatic Dispersion
Arises from the variation of refractive index with wavelength		Arises from the dependence of the fibre's waveguide properties on wavelength		

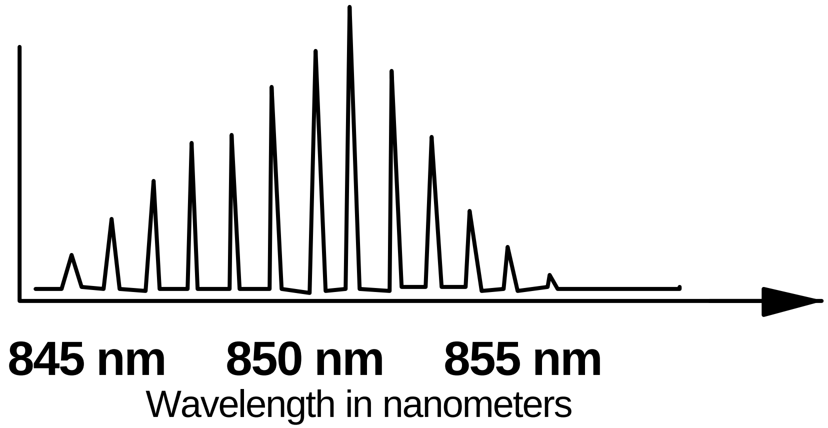
Material Dispersion

Material Dispersion Overivew

- Sometimes called Intramodal or Colour dispersion
- Results from the different group velocities of the various spectral components launched into the fibre by the source
- Typical optical source has an optical output that spreads over a range of wavelength.
- Spectral "width" can be defined as either an r.m.s value or a FWHM value



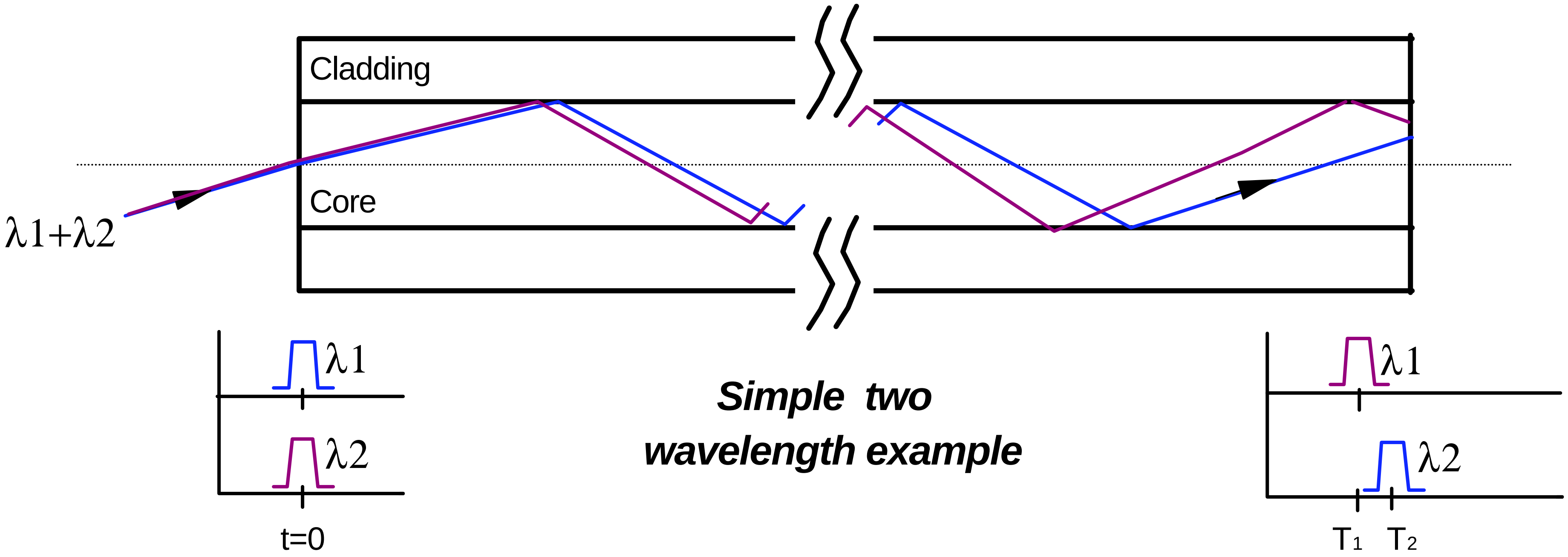
LED: Typical spectral width is 75-125 nm



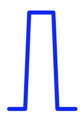
Conventional Laser:
Multimode operation
Typical spectral width 2-5 nm

Explaining Material Dispersion

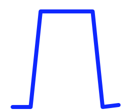
In an optical fibre the propagation velocity varies with wavelength. Thus a pulse made up of many wavelengths will be spread out in time as it propagates



Net Pulse Width at fibre input

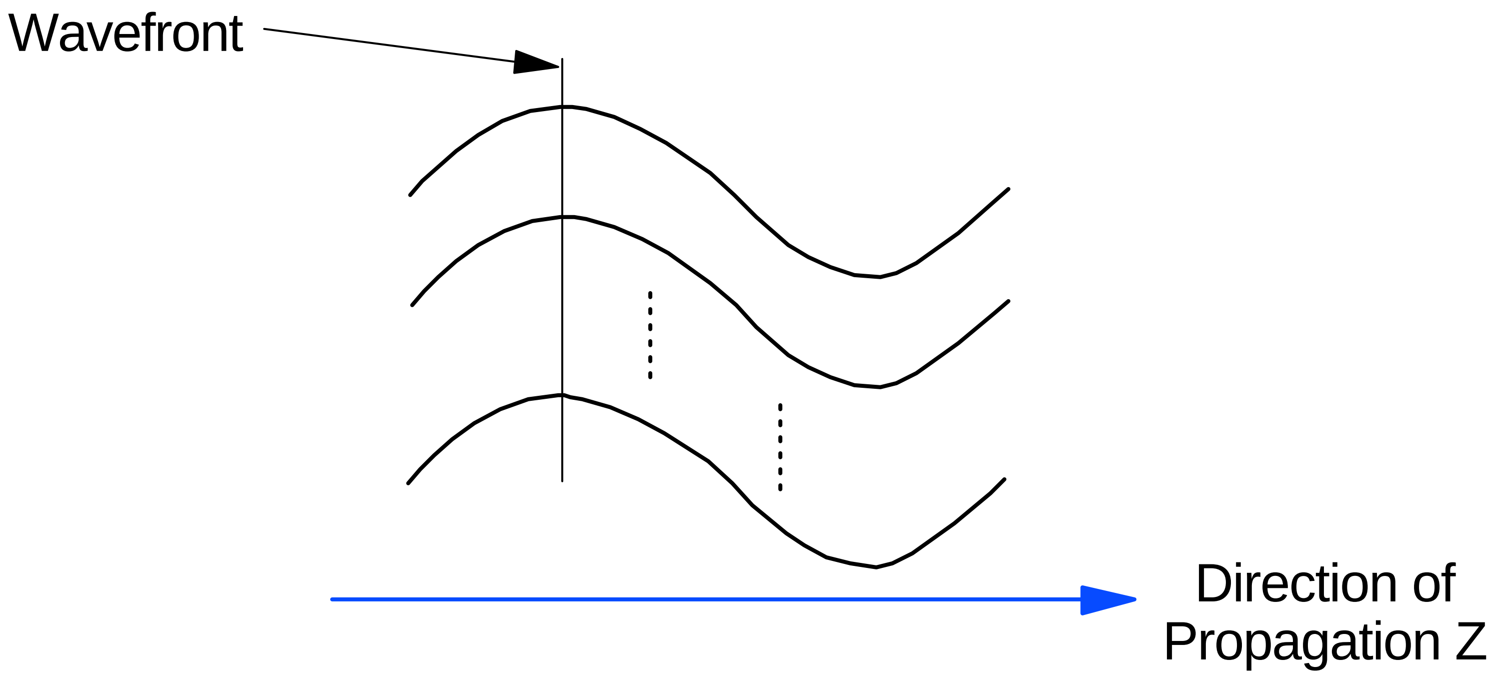


Net Pulse Width is approx $T_2 - T_1$



Analysing Material Dispersion

Phase Velocity (I)



- In an optical waveguide where a plane wave propagates the points of constant phase are called a "wavefront"
- For monochromatic light the points of constant phase propagate with a velocity called the "phase velocity $\boldsymbol{v}_p$ "

Phase Velocity (II)

- **If n is the refractive index of the medium, then the phase velocity as expected is:**

$$v_p = {}^{c}/_{n}$$

- **In free space we define the "free space" wavelength, λ as *c/f***
- **In medium of refractive index n > 1, the velocity changes and as frequency is a constant, we define the wavelength in the medium λ_m as λ/n**
- **As n > 1 then $\lambda_m < \lambda$**
- **The phase velocity v_p in a medium can also be written as:**

$$v_p = \lambda_m . f$$

Phase Velocity (III)

- **For a plane wave in a medium by convention we define the so-called propagation constant *b* thus:**

$$b = {}^{2p}/_{l_m}$$

- **The angular frequency w is *2p.f*, so the phase velocity can be written as:**

$$v_p = {}^{2p}/_{b} \,.f$$

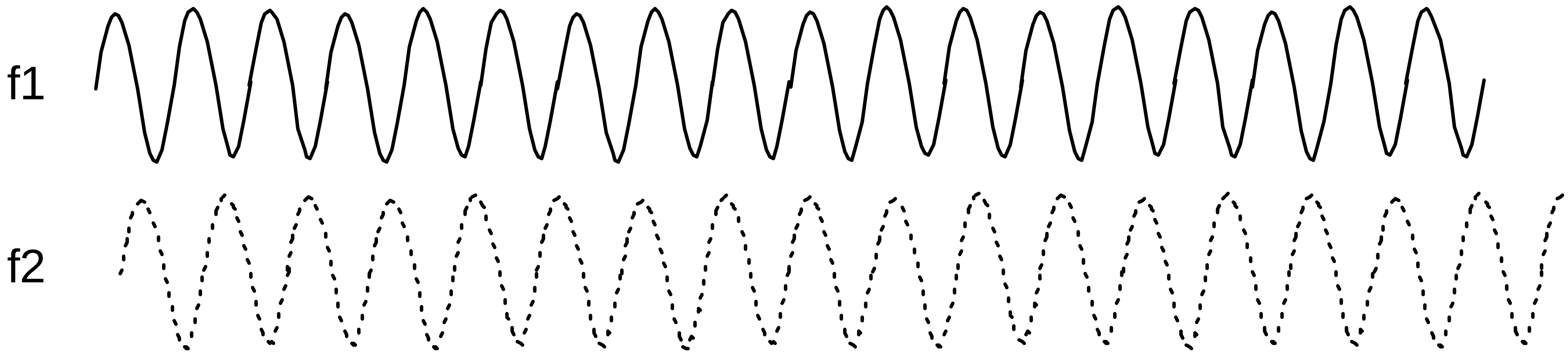
- **The phase velocity can thus be written as:**

$$v_p = {}^{w}/_{b}$$

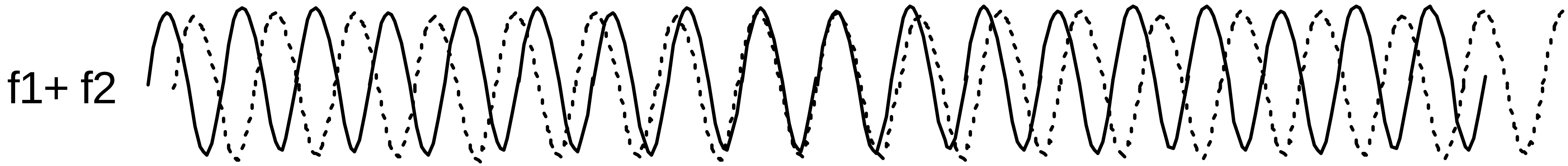
where w is the angular frequency and b is the propagation constant in the medium

Wavepackets and Group Velocity (I)

- Consider now two plane waves with nearly equal frequencies, f1 and f2

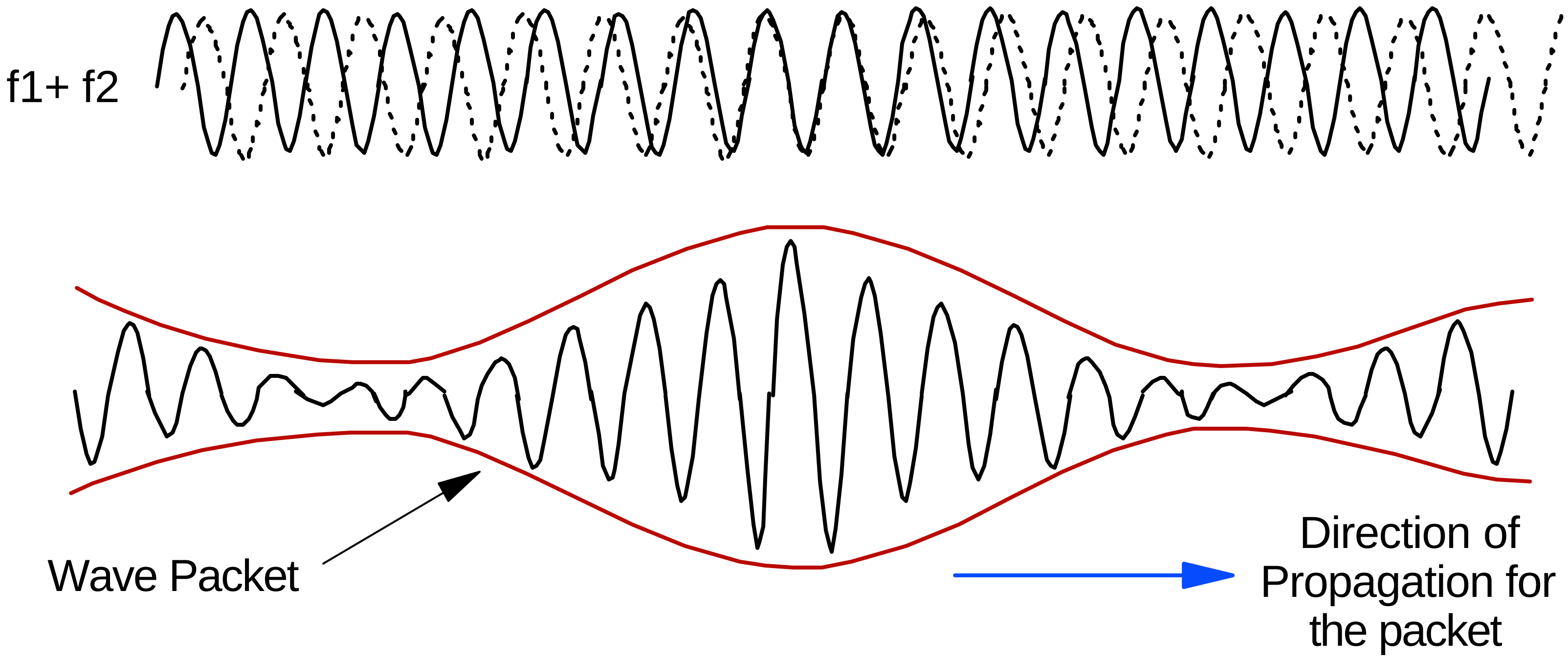


- If these plane waves propagate along the same medium then there are in-phase points and out-of-phase points thus:



Group Velocity (II)

- **The two plane waves with nearly equal frequencies, f1 and f2 when added form a wave packet or "group of waves"**
- **The wave packet propagates in the direction of travel of the plane wave**



Group Velocity (III)

- **If information is superimposed on the optical signal as a pulse then many packets of waves with closely similar frequencies propagate.**
- **Each wave packet propagates with a so-called group velocity v_g given by:**

$$v_g = {}^{dw}/_{db}$$

(Eq. 1)

where ω is the angular frequency and β is the propagation constant

- **Group velocity is the velocity of energy propagation through the system.**
- **Each wavepacket has its own propagation velocity and thus its own time delay.**

Group Velocity (IV)

- **Now**

$$v_g = {}^{dw}\!/_{db}$$

- **The time delay per unit length L of a medium, written as the group delay t_g can be shown to be given by:**

$$t_g = \frac{-l^2 L}{2p.c} \cdot {}^{db}\!/_{dl} \qquad \textbf{(Eq. 2)}$$

- **This equation is the starting point for a dispersion analysis**

Material Dispersion

- **In a medium that is susceptible to material dispersion, the refractive index is itself is a function of wavelength n(l).**

- **Thus the propagation constant *b* is a more complex function of wavelength. The nature of the dependence of *b* on wavelength will determine if dispersion (pulse broadening) takes place or not.**

- **For reference the so-called free space propagation constant *k* is given by $^{2p}/_{l}$**

- **The propagation constant in the medium is given by:**

$$b = k\,n(\lambda) = \frac{2p.n(l)}{l} \qquad \textbf{(Eq. 3)}$$

Material Dispersion

- Using equation (2) and (3) it is possible to determine the group delay as a function of wavelength and refractive index in a medium where refractive index is itself a function of wavelength
- Assume propagation distance L
- Group delay is given by:

$$t_g = \frac{L}{c}\left[n - l \frac{\mathrm{d}n}{\mathrm{d}l} \right] \qquad \textbf{(Eq. 4)}$$

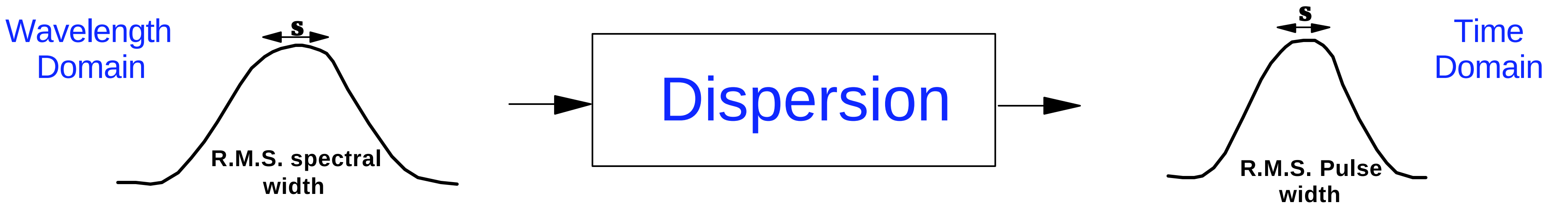
Analysis for Material Dispersion (I) in a fibre

Using equation (4) for an optical fibre core, the time t_m (fibre version of t_g to avoid confusion) taken for a pulse to propagate a distance L in a fibre is given by:

$$t_m = \frac{L}{c}\left[n_1 - l \frac{dn_1}{dl} \right] \qquad \textbf{(Eq. 5)}$$

If we have an impulse source with an RMS optical spectral width of s_l and a mean wavelength of l. then each spectral component will arrive at a different point in time so each t_m value will be different.

We want to determine the pulse broadening due to a spectral broadening



Analysis for Material Dispersion (II)

Assume a source with an rms optical spectral width of s_l and a mean wavelength of l. The rms pulse broadening in time due to material dispersion s_m may be found by expanding equation (5) using a Taylor series

$$s_m = s_l \frac{\mathrm{d}t_m}{\mathrm{d}l} + s_l \frac{2\,\mathrm{d}^2 t_m}{\mathrm{d}l^2} \cdots$$

In practice it is found that the first term normally dominates.

$$s_m = s_l \frac{\mathrm{d}t_m}{\mathrm{d}l} \qquad \textbf{(Eq. 6)}$$

The differential term is a problem. We need a term that contains measurable attributes such as refractive index and wavelength.

Analysis for Material Dispersion (III)

Now the first derivative of τ_m with respect to λ. can be found by differentiating equation (5) with respect to λ thus:

$$\frac{d\tau_m}{d\lambda} = \frac{L\lambda}{c}\left[\frac{dn_1}{d\lambda} - \frac{d^2 n_1}{d\lambda^2} - \frac{dn_1}{d\lambda}\right]$$

$$= \frac{-L\lambda}{c}\left[\frac{d^2 n_1}{d\lambda^2}\right] \qquad \textbf{(Eq. 7)}$$

Using equation (6) we can now write:

$$\sigma_m \cong \sigma_\lambda \frac{L}{c}\left| -\lambda \frac{d^2 n_1}{d\lambda^2}\right| \qquad \textbf{(Eq. 8)}$$

This is an important result. It is the rms spread of an impulse in time due to material dispersion after a distance L km. *Clearly if the second derivative is zero then dispersion is zero*

Quantifying Material Dispersion

To use equation (8), several helpful parameters have been defined and are available for a particular manufacturers fibre:

$$Y_m = -l^2 \frac{d^2 n_1}{d\,l^2}$$

Dimensionless dispersion coefficient

$$D_c = \frac{-l}{c}\left[\frac{d^2 n_1}{d\,l^2}\right]$$

So called material dispersion coefficient

units are ps /(km . nm)

So finally:

$$s_m \cong s_l \frac{L}{c.l}\left|Y_m\right| \quad \text{or} \quad s_m \cong s_l L\left|D_c\right|$$

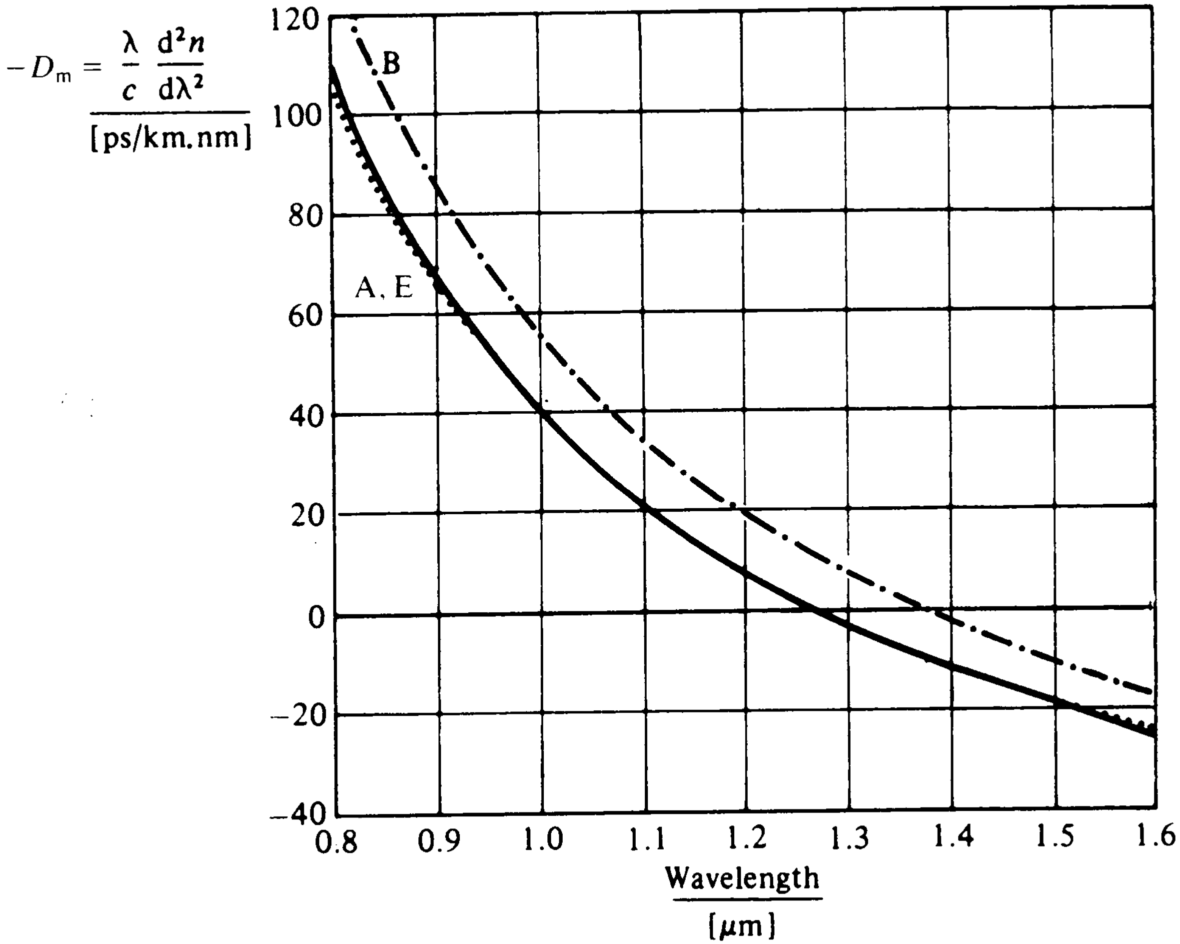
Material Dispersion Summary

- **Results from the different group velocities of the various spectral components launched into the fibre by the source**
- **In a dielectric medium the refractive index varies with wavelength.**
- **The velocity of propagation *v* varies with refractive index.**
- **The velocity of propagation varies with wavelength.**
- **If the variation in the refractive index with wavelength is nonlinear then dispersion takes place.**
- **The condition for non-zero dispersion is:**

$$\frac{d^2 n}{d\, l^2} \neq 0$$

Material Dispersion Parameter

- **The Material Dispersion Parameter D_m can be measured**
- **Graph shows D_m for various doped bulk silica samples**



Dispersion Problem

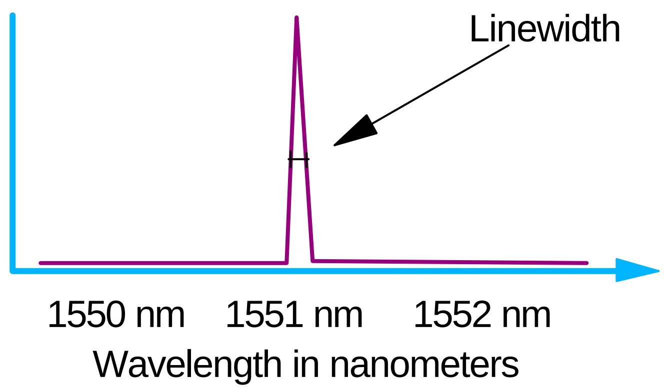
Problem: For a fibre with a Y_m value of 0.025, show that the material dispersion parameter is given by 98.1 ps per km.nm for a wavelength of 850 nm. Hence estimate the R.M.S. pulse spread at 850 nm for a LED source with an rms spectral width of 20 nm, assuming a 1 km long fibre.

Reducing Material Dispersion

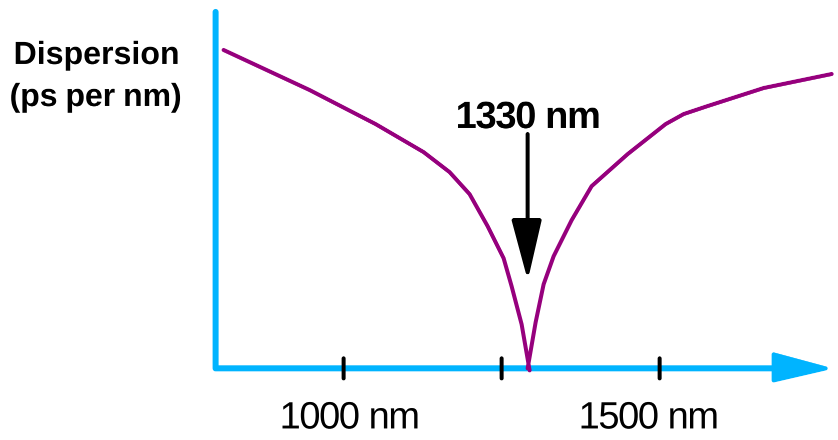
Use a singlemode laser with a narrow spectral width. For example a "Distributed feedback laser" (DFB) has a linewidth of about 10 - 30 MHz

Note: 1 GHz is approx 0.006 nm

DFB Laser Spectrum



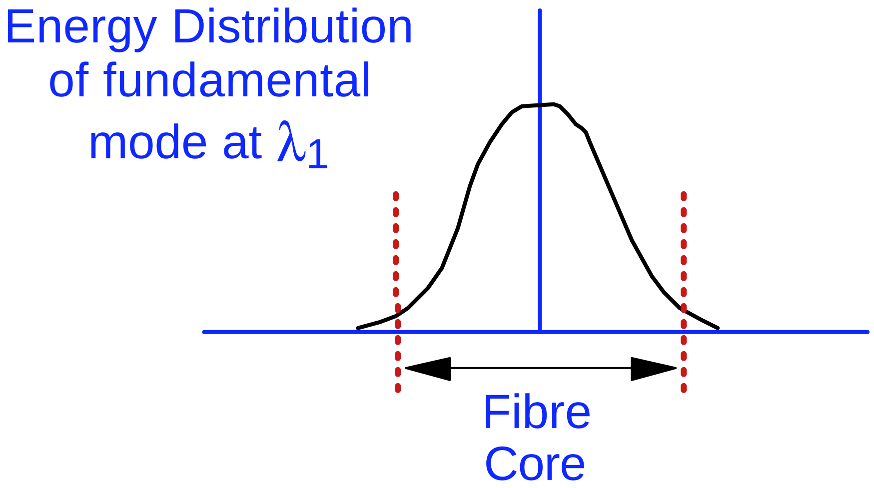
Operate at a wavelength with minimum material dispersion. Silica fibres have a natural region of negligible material dispersion around 1330 nm

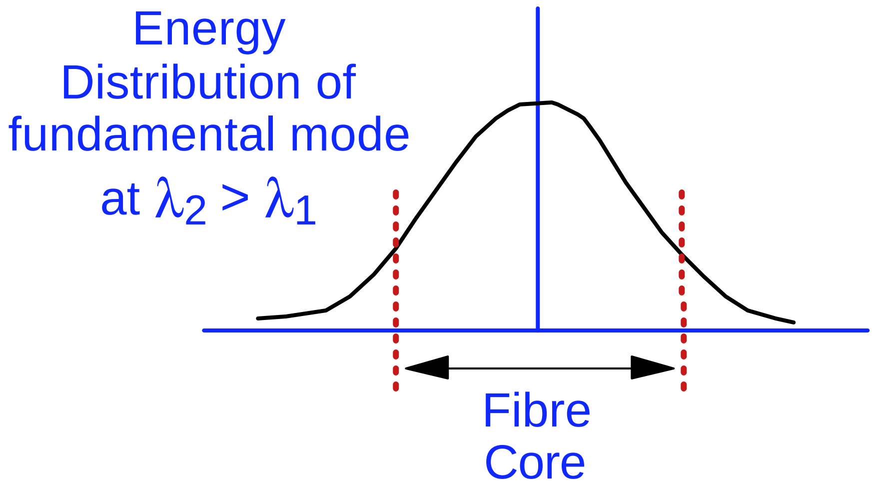


Waveguide Dispersion

What is Waveguide Dispersion?

- Caused by the wavelength dependence of distribution of energy for the fundamental mode in the fibre
- Mainly a problem for singlemode, in multimode mode penetration into the cladding is very small relatively.
- As wavelength increases an increasing proportion of the mode energy propagates in the cladding.
- But the cladding refractive index is lower thus faster propagation
- Thus for a spectral width σ_λ time delay differences (dispersion) develops





Waveguide Dispersion

- Mainly a problem for singlemode, in multimode mode penetration into the cladding is very small in a relative sense.
- Waveguide and material dispersion are controlled to give required overall chromatic dispersion
- Altering the refractive index profile will alter the waveguide dispersion
- Magnitude of waveguide dispersion is relatively independent of wavelength

Refractive index profiles

Conventional singlemode fibre (so called matched cladding)

Triangular profile singlemode fibre (used in dispersion shifted fibre)

Total Dispersion and Dispersion Comparisons

Dispersion Specifications

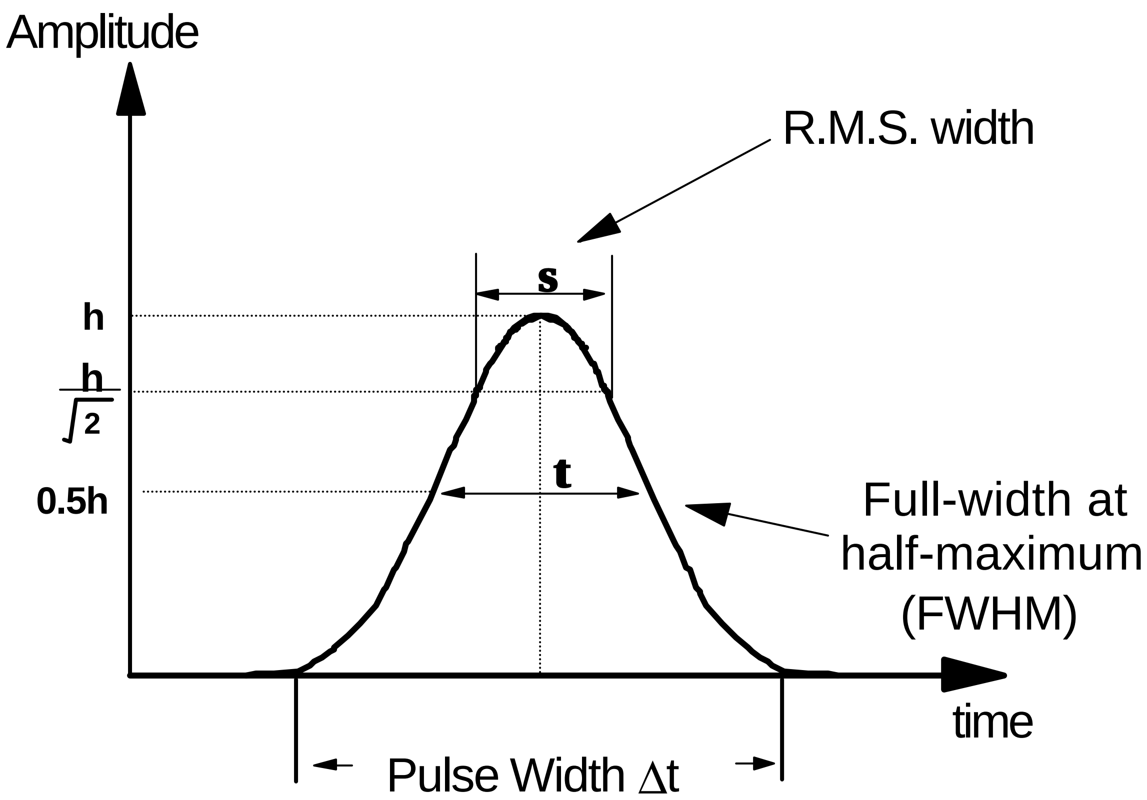
Multimode Fibre

- For multimode fibre dispersion is both chromatic and modal
- Dispersion rarely specified, instead fibre bandwidth is specified
- Maximum bit rate can be found from bandwidth

Singlemode Fibre

- For singlemode fibres chromatic dispersion is present (also PMD?)
- Chromatic dispersion is specified as ps/nm-km
- ITU defines limits for various types and wavelengths
- Total dispersion must be calculated

Pulse Shape Definitions



Some pulse types

Rectangular:

$t = Dt$ and $s = 0.289\ Dt$

Triangular:

$t = 0.5\ Dt$ and $s = 0.204\ Dt$

Gaussian:

$t = 2.35\ s$ and $s = 0.425\ t$

Note: For gaussian the pulse width Δt is infinite and thus has little practical meaning

Maximum Bit Rate

- Dispersion results in pulse broadening, reducing the maximum bit rate
- Several approximate rules of have evolved in the literature for relating dispersion to a maximum bit rate B_t :

$$B_t \le \frac{1}{\Delta t} \text{ bits / sec}$$

Simplest rule. Assumes that no I.S.I is allowed to take place so the Dt after dispersion must be less than the bit interval T. Since the bit rate is the reciprocal of the bit interval we get the rule shown. Also assumes impulse like input pulse shapes.

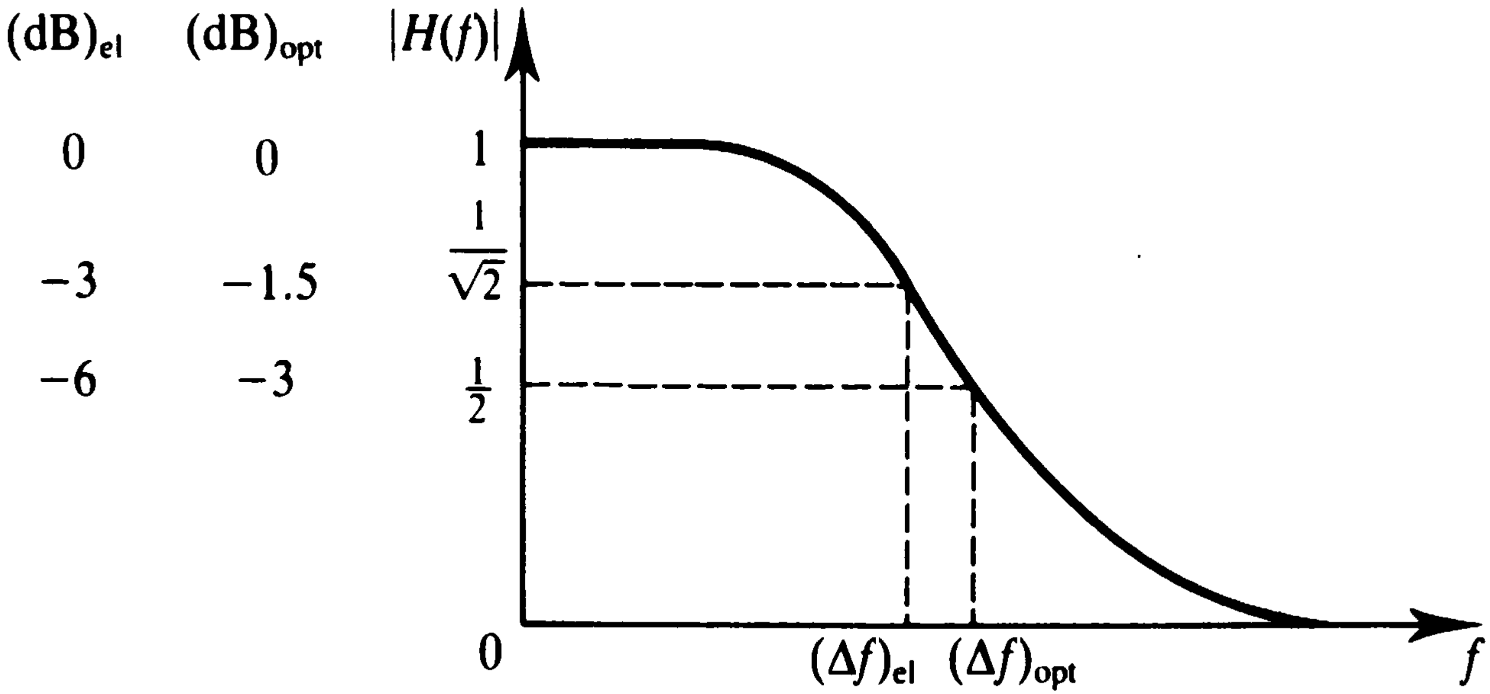
$$B_t \cong \frac{0.2}{s} \text{ bits / sec}$$

More realistic rule based, on the assumption that the broadened pulse has a gaussian shape, with an rms width of s.

Some sources will use a more optimistic 0.25 instead of the factor of 0.2

Optical .V. Electrical Bandwidth

- 3 dB optical bandwidth is the frequency on the fibre transfer function *H(f)* where the power is half the low frequency value
- In a receiver the electrical level at the optical 3 dB freq is down 6 dB (see diagram)
- *The Optical and Electrical 3 dB frequencies are not equivalent*



Bandwidth and Bit Rate

- Fibre bandwidth may be directly specified
- The maximum bit rate B_t and the fibre bandwidth are related
- Relationship depends on the transmitter pulse shape and the receiver equalisation employed
- 3 dB electrical (BW_e) and optical (BW_o) bandwidths are different

For rectangular pulse shapes:

$$B_t = 1.96 \times (BW_e) = 1.44 \times (BW_o)$$

For gaussian pulse shapes

$$B_t = 1.89 \times (BW_e) = 1.34 \times (BW_o)$$

Bandwidth-Length Product

- The longer the fibre length involved the greater the dispersion
- Dispersion or pulse broadening is specified as ns or ps per km
- Bandwidth-length product is a figure of merit for comparing different systems. Units are typically MHz.km or GHz.km

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Problem:

Two graded index fibre systems are to be compared. System A has a total dispersion of 6.5 ns over 15 km, while system B has total dispersion of 7.1 ns over 16 km.

Assume a realistic rule relating maximum bit rate and dispersion, a gaussian pulse shape at the fibre output and impulse like input pulses,

Which fibre has the higher bandwidth-length product?

Problem Solution

System A:

6.5 ns total dispersion.

Using the rule that $B_t = 0.2/\sigma$ bits/sec then the maximum bit rate is 30.7 Mbits/sec.

The optical bandwidth is 22.9 Mhz

The bandwidth-length product is 22.9 x 15 = 343.50 MHz.km

System B:

7.1 ns total dispersion.

Using the rule that $B_t = 0.2/\sigma$ bits/sec then the maximum bit rate is 28.2 Mbits/sec.

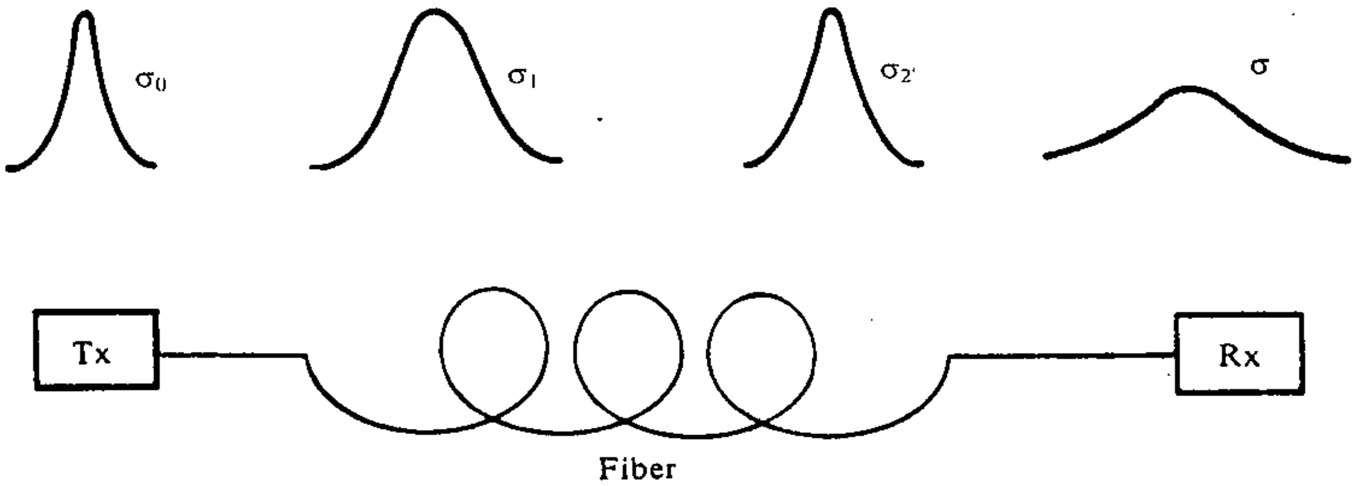
The optical bandwidth is 21.04 Mhz

The bandwidth-length product is 21.04 x 16 = 336.64 MHz.km

Conclusion: System A has the better bandwidth-length product

Total Dispersion

Transmitted pulse	Pulse Broadening due to chromatic dispersion	Pulse Broadening due to modal dispersion	Received pulse



<u>Received pulse width</u>

$$s = (s_o^2 + s_1^2 + s_2^2)^{1/2}$$

Assumes uncorrelated dispersion mechanisms and gaussian pulse shapes

ITU-T Fibre Recommendation G.652

- ITU-T Rec.G.652 for singlemode fibres:
- *Wavelength range circa 1310 nm*
  - Attenuation < 0.36 dB/km
  - Maximum dispersion 3.5 ps/(nm.km)
- *Wavelength range circa 1550 nm*
  - Attenuation < 0.25 dB/km
  - Dispersion 18 ps/(nm.km)

Note: Equivalent to the IEC-60793-1 standard

Finding the Total Chromatic Dispersion

Total Chromatic Dispersion = $D_c \times \sigma_\lambda \times L$

where:

D_c is the dispersion coefficent for the fibre (ps/nm.km)

σ_λ is transmitter source spectral width (nm)

L is the total fibre span (km)

- Assuming singlemode fibre so there is no modal dispersion
- Does not include polarization mode dispersion
- Typically the dispersion coefficent will be known
- Eg. ITU-T Rec.G.652 for singlemode fibres circa 1550 nm states:
  - Attenuation < 0.25 dB/km
  - Dispersion coefficent is 18 ps/(nm.km)

Total Chromatic Dispersion Example

- 50 km of singlemode fibre meeting ITU G.652
- 1550 nm DFB laser with a spectral width of 0.1 nm

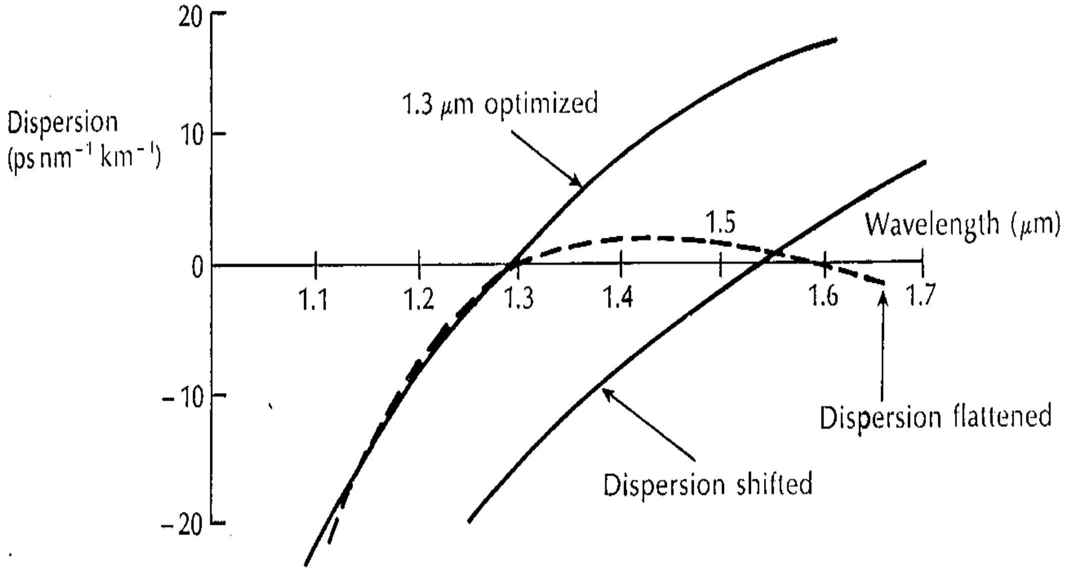
Total Dispersion = $D_c \times \sigma_\lambda \times L$

= 18 ps/nm.km x 0.1 nm x 50 km

= 90 ps total dispersion

Dispersion Shifted Fibre

- **A shift in the wavelength, λ_o, at which minimum dispersion is achieved is frequently desirable, i.e. if one wishes to operate at the wavelength of minimum attenuation (circa 1550 nm).**
- **Fine tuning the profile or the dopants used in the fibre can alter λ_o.**



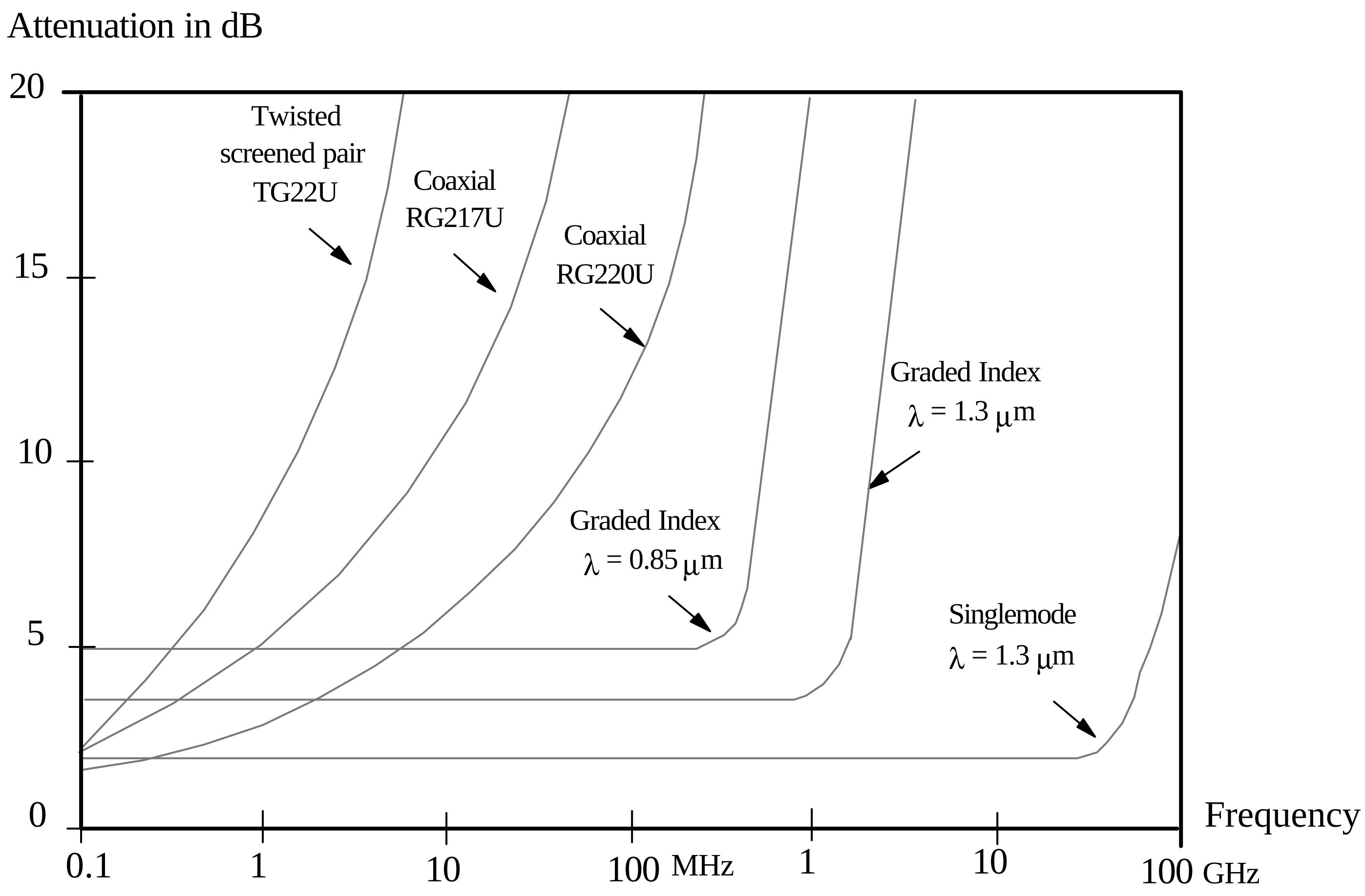
- **Implications for Dense Wavelength Division Multiplexed systems**

ITU-T Fibre Recommendations G.653 and G.655

- ITU-T Rec.G.653 for dispersion shifted singlemode fibres:
- *Wavelength range circa 1550 nm*
  - Attenuation < 0.25 dB/km
  - Dispersion 3.5 ps/(nm.km)

- ITU-T Rec.G.655 for non-zero dispersion shifted singlemode fibres (under study):
- *Wavelengths between 1530 and 1565 nm*
  - Attenuation < 0.25 dB/km
  - Minimum dispersion > 0.1 ps/(nm.km)
  - Maximum dispersion< 6.0 ps/(nm.km)

Comparison of Fibre and Copper Bandwidths



Supplied by Wavetek

Comparison of Fibre Bandwidths

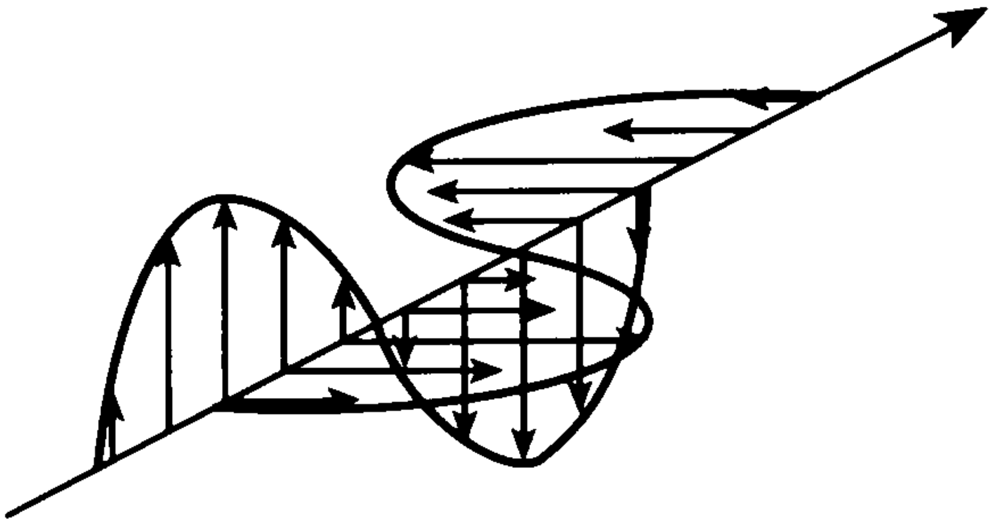
			Typical Actual Bandwidths in MHz.km			
Fibre Geometry Core/Cladding diameter in microns	NA	Potential Bandwidth in MHz.km	Laser @ 1330 nm	Laser @ 850 nm	LED @ 1330 nm	LED @ 850 nm
8/125	0.11	infinite	>10,000	*	*	*
50/125	0.20	2,000	1,000	400	600	200
62.5/125	0.275	1,000	500	160	400	80
100/140	0.29	500	300	100	250	50

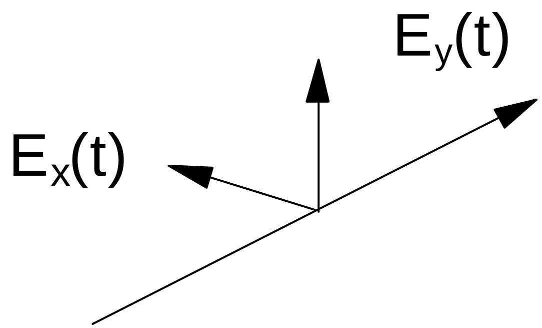
The symbol * indicates an unlikely choice

Polarization

Polarization in a Fibre

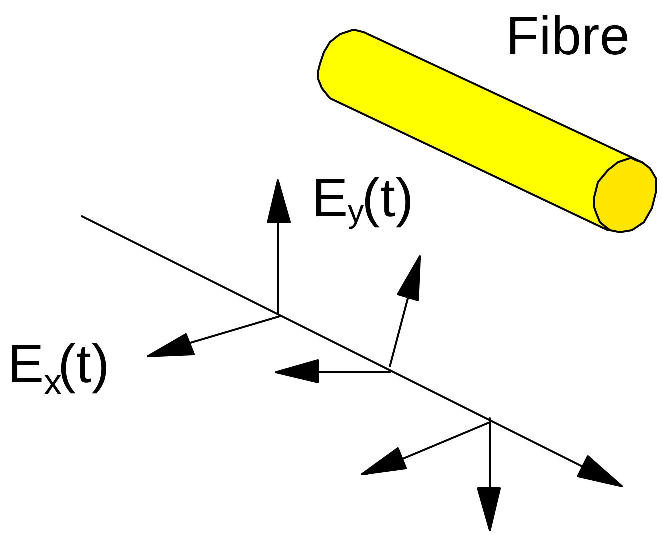
- Light is an electromagnetic field
- It always can be decomposed in two "polarizations",Ey(t) and Ex(t)
- Both polarizations are orthogonal to each other (at 90 degrees to each other)
- Also orthogonal to the direction of propagation
- Normally represented as arrows for simplicity





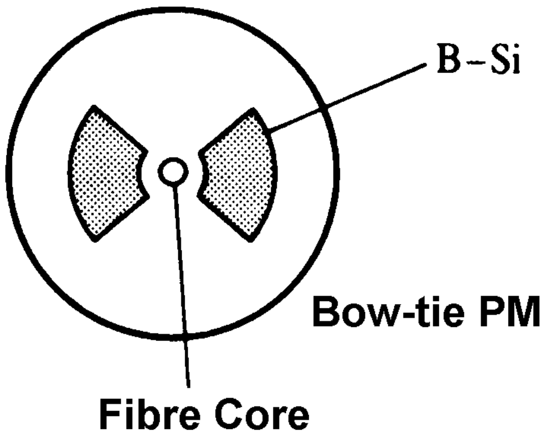
Changes in Polarization

- In normal fibre the polarization state (called the plane of polarization) changes randomly along the fibre
- Occurs because fibre is not a perfectly uniform medium and because of mechanical stress. This is called birefringence
- This is a problems for some types of advanced components which are sensitive to the state of polarization
- So called Polarization Maintaining Fibre (PMF) is available

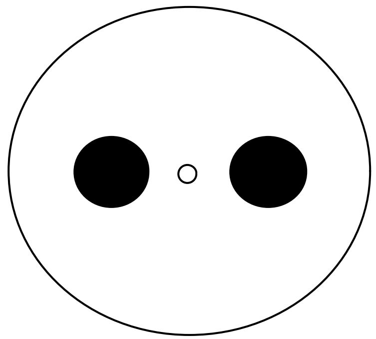


Polarization Maintaining Fibre

- So called Polarization Maintaining Fibre (PMF) is available
- Fibres are designed to have a specific internal stress that holds the plane of polarization
- Several varieties of PMF have been developed: Bow-tie and Panda
- In bow-tie PMF the cladding contains portions of boron doped silica glass
- Thermal expansion differences stress the fibre in a controlled manner



Bow-tie PMF



Panda PMF

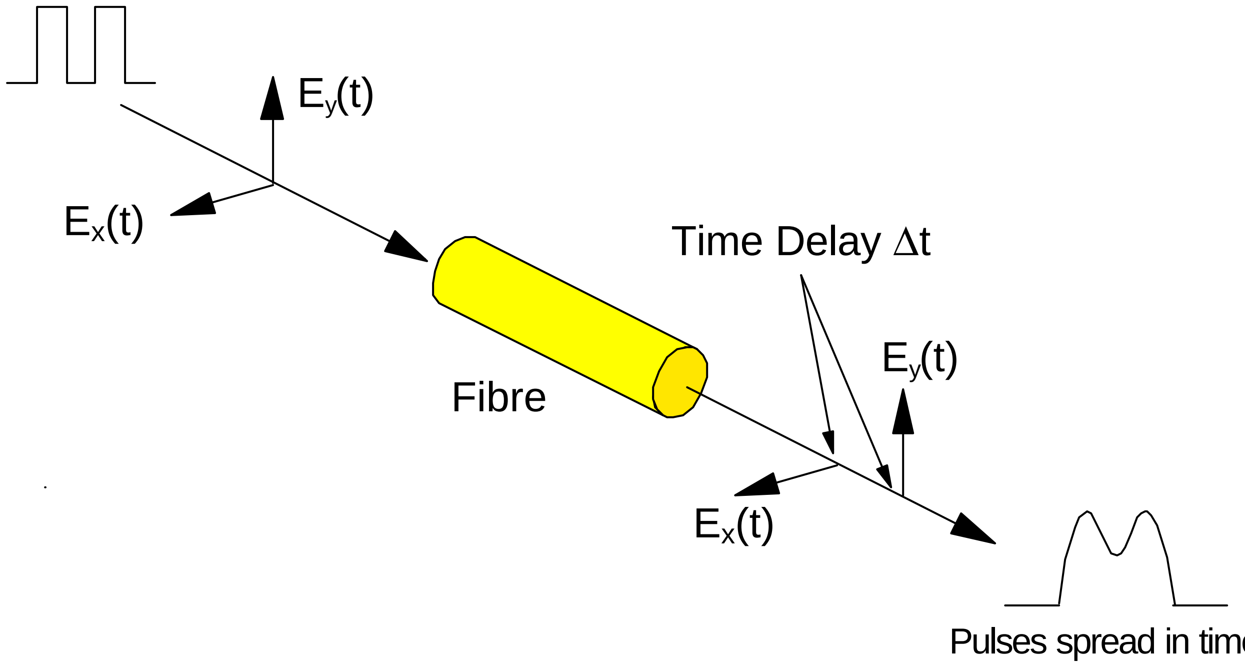
Polarization Mode Dispersion

Polarization Mode Dispersion

- Caused by cylindrical asymmetry due to manufacturing, temperature, bends, and so forth that lead to birefringence
- Input pulse excites multiple polarization components
- Pulse broadens as the polarization components travel at different speeds (disperse) along the fibre
- A key factor at bit rates above STM-16 (2.5 Gbits/sec), eg. at STM-64.
- Average value of PMD is well known.
- Instantaneous PMD varies unpredictably from the average, as a result it is difficult to compensate for.
- Methods of compensating for PMD in development

Polarization Mode Dispersion Principles

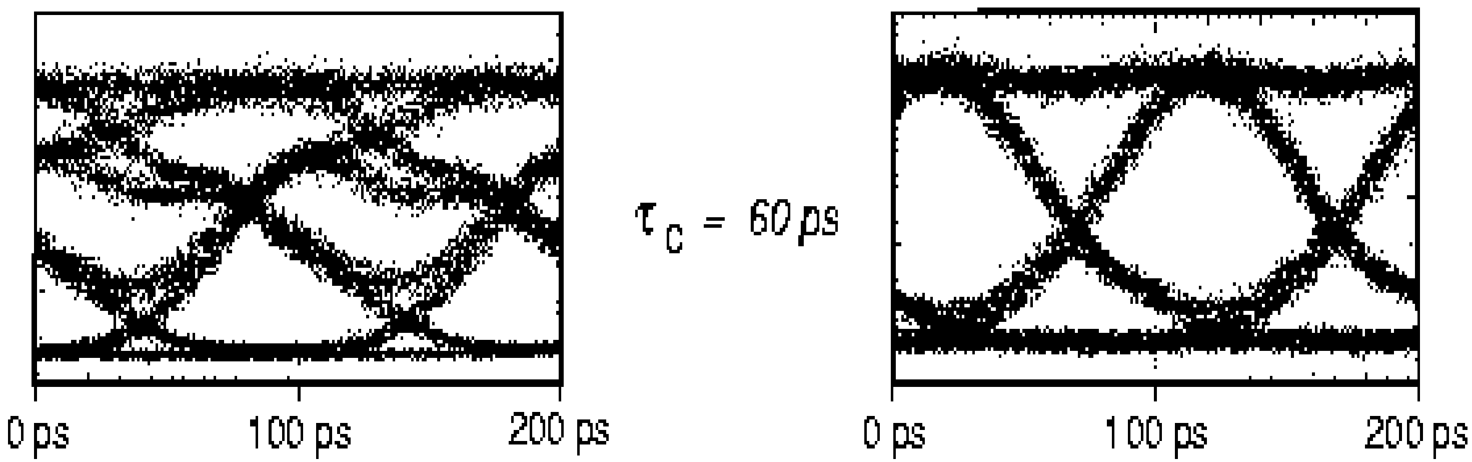
- Over a length of fibre the polarization states travel at different speeds
- In effect the states become unsynchronised
- Result is that signal energy reaches the fibre end at different point in time
- This causes pulse spreading or dispersion



Example of PMD

Eye Diagram (10 Gb/s)
with PMD

Eye Diagram (10 Gb/s)
without PMD



Polarization Mode Dispersion Limits

- PMD increases with the square root of distance
- Units are ps per root-km
- Limits normally specified for a 1 dB PMD penalty in the power budget

	Bit rate	Maximum PMD	PMD coefficient for 400 km link (ps/km$^{1/2}$)
STM-16	2.5 Gbits/s	40 ps	≤ 2
STM-64	10 Gbits/s	10 ps	≤ 0.5
STM-256	40 Gbits/s	2.5 ps	≤ 0.125

Recommended PMD levels for a 1 dB penalty