Channel Capacity

Lecture No. 11

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Overview

These lectures will look at the following:

- Channel capacity
- Shannon's channel capacity theorem
- Capacity versus bandwidth

Introduction

- Channel capacity is concerned with the information handling capacity of a given channel. It is affected by:
  - The attenuation of a channel which varies with frequency as well as channel length.
  - The noise induced into the channel which increases with distance.
  - Non-linear effects such as clipping on the signal.

  Some of the effects may change with time e.g. the frequency response of a copper cable changes with temperature and age. Obviously we need a way to model a channel in order to estimate how much information can be passed through it. Although we can compensate for non linear effects and attenuation it is extremely difficult to remove noise.

The **highest rate of information** that can be transmitted through a channel is called the **channel capacity, C**.

Shannon's Channel Coding Theorem

- **Shannon's Channel Coding Theorem** states that if the information rate, R (rH bits/s) is equal to or less than the channel capacity, C, (i.e. $R < C$) then there is, in principle, a coding technique which enables transmission over the **noisy** channel with no errors.
- The inverse of this is that if $R > C$, then the probability of error is close to 1 for every symbol.
- The **channel capacity** is defined as:
  the maximum rate of reliable (error-free) infor-

mation transmission through the channel.

Shannon's Channel Capacity Theorem

- **Shannon's Channel Capacity Theorem** (or the Shannon-Hartley Theorem) states that:

$$C = B\log_2\left(1+\frac{S}{N}\right) \quad bits/s$$

where C is the channel capacity, B is the channel bandwidth in hertz, S is the signal power and N is the noise power (N_0B with $N_0/2$ being the two sided noise PSD). **Note:** S/N is the ratio watt/watt not dB.

- The channel capacity, C, increases as the available bandwidth increases and as the signal to noise ratio increases (improves).
- This expression applies to information in any format and to both analogue and data communications, but its application is most common in data communications.
- The channel capacity theorem is one of the most important results of information theory. In a single formula it highlights the interplay between 3 key system parameters:
  - channel bandwidth,

  - average transmitted or received signal power,
  - noise power at the channel output.

- For a given average transmitted power S and channel bandwidth, B, we can transmit information at the rate C bits/s with no error, by employing sufficiently complex coding systems. It is not possible to transmit at a rate higher than C bits/s by any coding system without a definite probability of error. Hence the channel capacity theorem defines the fundamental limit on the rate of error-free transmission for a power-limited, band-limited channel.

Sample Question 1

A telephone network has a bandwidth of 3.4 kHz.

(a) Calculate the capacity of the channel for a signal-to-noise ratio of 30 dB.

(b) Calculate the minimum signal-to-noise ratio required for information transmission through the channel at the rate of 4800 bits/s.

(c) Calculate the minimum signal-to-noise ratio required for information transmission through the channel at the rate of 9600 bits/s.

Capacity versus Bandwidth

- It appears from the expression:

$$C = B\log_2\left(1+\frac{S}{N}\right) \quad bits/s$$

that as the bandwidth increases the capacity should increase proportionately. But this does not happen, because increasing the bandwidth, B, also increases the noise power $N = N_0 B$ giving:

$$
\begin{aligned}
C &= B \log_2 \left(1 + \frac{S}{N}\right) \\
&= B \log_2 \left(1 + \frac{S}{N_0 B}\right) \\
&= \frac{S}{N_0} \frac{N_0 B}{S} \log_2 \left(1 + \frac{S}{N_0 B}\right) \\
&= \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 B}\right)^{\frac{N_0 B}{S}}
\end{aligned}
$$

Consider the case where an infinite bandwidth is available. Increasing B to ∞ means that $S/N_0B \rightarrow 0$. The expression $\lim_{x\rightarrow 0}(1+x)^{1/x} = e$. This means that as the bandwidth goes to infinity, S/N_0B goes to 0 and $(1+S/N_0B)^{N_0B/S}$ goes to e. The channel capacity therefore goes to:

$$
\begin{aligned}
\lim_{B\rightarrow\infty} C &= \lim_{B\rightarrow\infty} \frac{S}{N_0} \log_2 \left(1 + S/N_0B\right)^{N_0B/S} \\
&= \frac{S}{N_0} \log_2 e \\
&= 1.44 \frac{S}{N_0}
\end{aligned}
$$

So as the bandwidth goes to infinity the capacity goes to $1.44S/N_0$, i.e., it goes to a finite value and is not infinite!

Sample Question 2

A communications channel with a bandwidth of 4 kHz has a signal power to noise ratio of 7. The bandwidth is reduced by 25 %. How much should the signal power be increased to maintain the same channel capacity?

Conclusion

These lectures have looked at the following:

- Channel capacity
- Shannon's channel capacity theorem
- Capacity versus bandwidth